

Epistemology Formalized

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As we often tell our undergraduates, epistemology is the study of *knowledge*. Given just this simple definition, ‘formal epistemology’ seems like a misnomer for the philosophical program inspired by Thomas Bayes and developed in the twentieth century by Ramsey (1926), de Finetti ([1937] 1993), Jeffrey (1983), and others. Bayesians articulate constraints on rational credences: synchronic constraints on what credences you may have, and diachronic constraints on how your credences must evolve. Like traditional epistemologists, Bayesians are concerned with norms governing your doxastic state. But in modeling your doxastic state, Bayesians do not represent what full beliefs you have.¹ And so they do not have the resources to talk about which of those beliefs constitute knowledge.

This paper develops a formal extension of traditional epistemology for which ‘epistemology’ is not a misnomer. I accept the traditional claim that beliefs can constitute knowledge. But I argue for an apparently radical thesis about the doxastic states that Bayesians care about: some of these states can also constitute knowledge. For example, suppose you are playing an ordinary poker game, and you have just been dealt some middling cards face down. Your justifiedly low credence that you have been

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1. At most, one might be able to derive facts about what full beliefs you have from facts that Bayesians do represent. But even this “Lockean Thesis” is contentious; see Foley 1993 and Huber and Schmidt-Petri 2009 for further discussion.

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dealt four aces may constitute knowledge. The thesis appears radical because knowledge is ordinarily assumed to be a factive attitude, where an attitude is *factive* if and only if necessarily, one bears it only to truths. Since your credence assignment is not truth-apt, this assumption seems to entail that your credence cannot constitute knowledge. The aim of this paper is to set out the strongest possible case for the apparently radical thesis in light of this simple argument against it. I argue that the thesis ultimately does not challenge our core assumptions about the nature of knowledge. And I argue that the thesis yields simple solutions to some difficult problems.

In section 1, I present the first of these problems: giving a semantics for factive attitude reports embedding language of subjective uncertainty. In section 2, I present a second problem: identifying what is wrong with the credences of agents in probabilistic analogues of Gettier cases. In section 3, I state my thesis and demonstrate that it can solve these problems. In section 4, I argue that the thesis is not as radical as it appears, since several core epistemological notions naturally extend to states other than full beliefs. Even if knowledge is not always a relation to propositions, there is nevertheless a sense in which it may be factive, safe, and sensitive. In section 5, I flag several decision points in the development and application of the notion of nonpropositional knowledge. I conclude in section 6 by outlining a number of further problems that may be solved by accepting nonpropositional knowledge, including problems that have been recently raised by opponents of pragmatic encroachment and knowledge-based norms for action.

1. The First Problem: Ascriptions Embedding Language of Subjective Uncertainty

The language of subjective uncertainty is pervasive in ordinary conversation:

- (1) John might be in his office.
- (2) Paris is probably the largest city in France.
- (3) Marseille is probably not the largest city in France.
- (4) If the second-largest British city is not Leeds, then it is probably Birmingham.
- (5) John must have gone home early today.
- (6) If John is still in the building, he is in his office.
- (7) It is more likely that John is in London than that he is in Paris.

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And yet it is notoriously difficult for traditional semantic theories to model such language. For instance, Kratzer 1977 gives a powerful truth-conditional semantics for necessity and possibility modals. Kratzer 1981a extends the account to some “graded” modal vocabulary, and Kratzer 1978 and 1981b discuss conditionals. On the theory that Kratzer develops in these papers, sentences containing language of subjective uncertainty are context sensitive. In particular, modals quantify over a contextually supplied domain of possibilities. This approach provides an elegantly uniform treatment of different modalities: *epistemic* modals are just those that quantify over *epistemic* possibilities.² However, many have raised serious objections to this prevailing truth-conditional theory of expressions of subjective uncertainty.

For instance, Yalcin (2007) argues that standard truth-conditional theories of epistemic modals fail to predict the behavior of embedded modals. For example, each of the following sentences is infelicitous:

- (8) #Suppose that it is not raining, and it might be raining.
- (9) #Suppose that it is raining, and it is probably not raining.
- (10) #If it is not raining and it is probably raining, then ...

According to standard truth-conditional theories, Yalcin argues, ‘it might be raining’ is true just in case certain contextually determined evidence does not rule out that it is raining. It is perfectly acceptable to suppose both that it is not raining and that certain evidence does not rule out that it is raining. So standard truth-conditional theories incorrectly predict that it is acceptable to suppose as in (8).

A second objection is that truth-conditional theories flout our intuitions about the subject matter of sentences containing language of subjective uncertainty. For example, consider the famous Sly Pete example from Gibbard 1981. Suppose you observe Sly Pete advance to the last round of a poker game. Just then you leave the room, but only after seeing that the unscrupulous Pete has looked at his opponent’s hand. On this basis, you may utter the following:

- (11) If Pete called, he won.

Kratzer (1986) says that in order to account for Gibbardian stand-offs, truth-conditional theories must claim that you use (11) to report facts about your own evidential state. But many have rejected this result as

2. For defenses of related contextualist accounts in the philosophical literature, see DeRose 1991 and its precedents Moore 1962, Hacking 1967, and Teller 1972.

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intuitively unacceptable. For instance, Bennett (2003, 90) provides the following argument about a close variant of the Sly Pete case: “Common sense and the Ramsey test both clamour that [the speaker of (11)] is not assuring me that her value for a certain conditional probability is high, but is assuring me of that high value. . . . She aims to convince me of that probability, not the proposition *that* it is her probability.” Yalcin (2009) adds that the *reasons* that I give in support of my utterance ‘it is probably raining’ concern only the proposition that it is raining, rather than any contextually determined body of evidence.

A third objection follows from an observation in Adams 1965 and Stalnaker 1970, namely, that your degree of belief in an indicative conditional should match your degree of conditional belief in the consequent given the antecedent. Unfortunately for the truth-conditional semanticist, the triviality results of Lewis 1976 demonstrate that indicative conditionals cannot express propositions such that your credence in the conditional proposition matches your conditional credence in the consequent given the antecedent. Many have responded by rejecting the claim that indicative conditionals express propositions.³

These objections merely scratch the surface. For additional arguments against truth-conditional theories, see the case of the missing car keys in Swanson 2006 and in von Fintel and Gillies 2011, the “speaker inclusion constraint” in Egan, Hawthorne, and Weatherson 2005 and in Weatherson 2008, the eavesdropping cases in Egan 2007, the discussion of assertability and disagreement in Yalcin 2011, and the discussion of retraction and disputes in MacFarlane 2011. In response to such arguments, many have developed alternative semantics for language of subjective uncertainty. Extant accounts include dynamic, relativist, and expressivist theories. On many of these accounts, (1)–(7) do not have straightforward truth conditions.⁴

3. For sympathetic discussion and a catalog of relevant literature, see Edgington 1995. An alternative interpretation understands Adams 1965 as defending a claim about appropriateness of utterances rather than degrees of belief. Hall and Hájek 1994 provides a helpful assessment of interpretations of Adams’ Thesis, as well as a catalog of further triviality results.

4. For relativist theories, see Egan, Hawthorne, and Weatherson 2005, Egan 2007, Stephenson 2007, Weatherson 2008, and MacFarlane 2011. For dynamic theories, see Veltman 1985, 1996; and Gillies 2004, 2009. For expressivist theories and cousins of expressivist theories, see Adams 1975, Edgington 1995, Bennett 2003, Swanson 2006, Yalcin 2007, Schnieder 2010, and Moss, n.d. For decisive arguments against force modifier theories, see Swanson 2010.

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Arguing for any particular semantic theory would take us too far afield of the present project. For ease of exposition, I shall assume a very simple expressivist semantics in my discussion of language of subjective uncertainty, a semantics along the lines of that defended in Swanson 2006 and Yalcin 2007. On this naïve semantics, assertions of sentences embedding language of subjective uncertainty express advice concerning credal states. In particular, the semantic value of a sentence is a constraint on your credence distribution, and an assertion of the sentence expresses the advice that your credence distribution conform to that constraint. The resulting semantics fits neatly with Bayesian doctrine: the semantic value of a sentence is a set of probability measures, and an assertion expresses the advice that your credence distribution be among the members of that set. For example:

- (3) Marseille is probably not the largest city in France.
- (4) If the second-largest British city is not Leeds, then it is probably Birmingham.
- (7) It is more likely that John is in London than that he is in Paris.

(3) advises you to give low credence to the proposition that Marseille is the largest city in France. (4) advises you to give high conditional credence to the proposition that the second-largest British city is Birmingham, conditional on the proposition that it is not Leeds. (7) advises you to give more credence to the proposition that John is in London than to the proposition that he is in Paris.

Expressivist theories provide a very natural account of attitude ascriptions embedding language of subjective uncertainty. A belief ascription simply says that the credences of the subject conform to the constraint that is the semantic value of the prejacent. For example, (12) says that Bob gives more credence to the proposition that John is in London than to the proposition that he is in Paris:

- (12) Bob believes that it is more likely that John is in London than that he is in Paris.

On this account, some attitude ascriptions ascribe relations not to propositions, but to constraints on probability measures. This fits nicely with the Bayesian claim that constraints on probability measures characterize the contents of some of our attitudes.

This natural account of attitude ascriptions constitutes yet another argument against the semantics for epistemic modals advocated in Kratzer 1977 and 1981a. As Yalcin 2011 points out, the way that Baye-

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sians informally describe credal states suggests that Bayesians already tacitly accept this sort of account of attitude ascriptions. For instance, Bayesians use (12) to describe credences in first-order propositions about John, not to describe beliefs in propositions about the likelihood of first-order propositions given a certain body of evidence. In fact, the expressivist account of ascriptions is so intuitive that advocates of truth-conditional theories have aimed to replicate its verdicts. For instance, Kratzer 2010 develops a semantics according to which (13) and (14) do not straightforwardly ascribe relations to propositions:

- (13) Bob believes that John might be in his office.
- (14) Bob believes that John must have gone home early today.

On the traditional truth-conditional semantics, (13) says that in every world compatible with what Bob believes, a certain body of evidence does not rule out that John is in his office. On the revised semantics in Kratzer 2010, (13) simply says that in some worlds compatible with what Bob believes, John is in his office. (14) says that in all worlds compatible with what Bob believes, John went home early.

The non-truth-conditional innovations I have described constitute major progress in the semantics for language of subjective uncertainty. But they also give rise to a serious problem. The problem arises because language of subjective uncertainty is not only commonly embedded in belief reports, but in reports of knowledge and other factive attitudes. For example:

- (15) The Fellahs advanced till they saw that it was probably a large tomb (Belzoni 1820, 231).
- (16) If you give a clear, understandable direction . . . and the child does not comply, then you know that it is more likely due to compliance issues than lack of understanding (Webb et al. 2007, 113).
- (17) I couldn't figure out how they had the entire day free to go tracking down Seth until I remembered it was most likely a Sunday (Comocrush 2009).
- (18) By monitoring your home a couple of times a year, you can observe fading and cracking issues, which lets you know that it is probably getting close to that time again (Hostetter 2012).

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- (19) I hereby let you know that it is more likely that your specimens belong to *G. hackmani* than to *G. balachowskyi* (Willem 2005).
- (20) I realized that I probably liked that guy as more than a friend. (*peedyCakey)

For the expressivist, the semantic value of the prejacent of the attitude report in (19) is the set of all probability measures that give higher credence to the proposition that the specimens under discussion are *G. hackmani* than to the proposition that they are *G. balachowskyi*. The attitude report ascribes a relation to that semantic value. But that spells trouble, for it is traditionally taken as a platitude that knowledge is an attitude that one can bear only to true propositions.⁵ Hence our first problem: knowledge is an attitude toward truths; (19) ascribes knowledge; yet (19) does not ascribe an attitude toward truths.⁶

The same problem arises for any theory that says that attitude reports ascribe attitudes to the contents of their embedded clauses, and that sentences containing expressions of subjective uncertainty do not have truth-conditional contents. In short, nearly everyone faces some form of the problem, including advocates of dynamic, relativist, and expressivist semantic theories, as well as Kratzer (2010). Our traditional understanding of knowledge does not fit our contemporary understanding of assertion and the ascription of attitudes.

2. The Second Problem: Probabilistic Analogues of Gettiered Beliefs

A second problem accompanies a contemporary formal understanding of learning. The following story is a familiar one: according to tradition, you should update your credence distribution by conditionalizing it on the propositions you learn. But some instances of learning intuitively call

5. I focus on knowledge-that rather than objectual knowledge, knowledge-how, or knowledge-wh. Ascriptions of the latter attitudes raise interesting issues that are largely orthogonal to the present discussion.

6. Here I use '(19)' to refer to the token utterance cited above. It is important to appreciate that a sentence containing language of subjective uncertainty may have multiple readings, including readings for which a contextualist semantics is appropriate. Anticontextualist arguments do not establish that no utterance containing language of subjective uncertainty expresses a proposition about a contextually determined body of evidence, but merely that not all utterances fit this mold. I discuss particular sentences for ease of exposition, but my arguments ultimately concern embedded language of subjective uncertainty not accommodated by contextualist accounts.

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for other updating procedures. Jeffrey 1965 argues that seeing a cloth in dim light may lead you to rationally assign .7 credence to the proposition that the cloth is green, and van Fraassen 1980 suggests that the soldier Judy Benjamin may directly update her conditional credences upon hearing sentences such as:

- (21) If you are in Red Army territory, you are .75 likely in their Headquarters Company Area.

On the resulting picture, you should update your credence distribution by adopting credences that conform to constraints delivered by your experiences. Joyce (2005, 158) sums up the picture as follows: “At any time there should be some set of constraints that specify those invariant features of a person’s credal state that are directly imposed by her evidence.” For van Fraassen (1981, 375), these features include “deliverances of experience” that constrain the credences that you can rationally have. Indeed, it is intuitively evident that experience can impose constraints on the credences you have in all the same ways that it can impose constraints on the propositions that you believe. The examples (15)–(20) given in the previous section are naturally understood as statements about constraints on credences provided by perception, inference, memory, testimony, and introspection, respectively.

It is notoriously difficult to defend general procedures for directly updating credences on constraints.⁷ But it is also necessary to appreciate the powerful reasons for endorsing alternatives to strict conditionalization. It is not just that our intuitions about particular cases call for alternative procedures. Strict conditionalization results in certainties, and certainties make agents prefer bets at arbitrarily risky odds and maintain opinions that cannot be altered by further learning. It may simply seem unreasonable for ordinary agents to have this degree of “blind faith” in propositions. This point is recognized as early as Quine 1951, where Quine argues that even observational propositions can be undermined by theoretical considerations. Even without endorsing any Quinean confirmational holism, many contemporary theorists accept that ordinary agents should not be perfectly certain of nontrivial propositions. Learning should not by its very nature require an agent to become dogmatically inflexible with respect to some opinion.

7. For further discussion, see Diaconis and Zabell 1982, Jaynes 1978, Skyrms 1987, Joyce 1999, and Grünwald and Halpern 2003.

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Just as beliefs that result from experience can be justified or unjustified, so can properties of your credence distribution. This claim is not meant to be surprising; others have defended the similar claim that your having a particular degree of belief in a proposition may or may not be justified.⁸ The present claim is simply that even if learning proceeds by directly updating on constraints—rather than by acquiring a special epistemic relation to individual propositions—resulting properties of your credence distribution may be justified. In other words, you may be justified in having credences with the property of satisfying the constraint on which you updated. As with full belief, this justification comes in degrees. Even if your experience justifies your giving .3 credence to a given proposition, you may gain further justification for that same credence through further experience. Judy Benjamin may call helicopter pilots to confirm the testimony of her first informant, for instance. Or she may spot flags on nearby trees, and recall that such markers are exactly three times as prevalent in the Red Army Headquarters Area as in the rest of the Red Army territory.

There are many perspicuous parallels between justified beliefs and justified properties of credence distributions. But not every parallel between beliefs and properties of credence distributions is so easily understood. In particular, there are cases in which justified properties of credence distributions fail to be epistemically good, and they fail in just the same way that justified beliefs fail to constitute knowledge. Recall the following sample ascriptions involving constraints on credences provided by testimony and introspection:

- (19) I hereby let you know that it is more likely that your specimens belong to *G. hackmani* than to *G. balachowskyi*.
- (20) I realized that I probably liked that guy as more than a friend.

Here are concrete examples where similar testimony and introspection intuitively fail to yield knowledge:

Several assistants at a reputable entomology supply company select specimens from cultures in response to mail order requests for cereal flies. There are two cultures, one consisting mainly of *G. hackmani* specimens and one consisting mainly of *G. balachowskyi* specimens. A diligent lab assistant sends out specimens from the former culture along with a letter to the recipient saying that their specimens are more likely *G. hackmani*

8. For some representative examples, see Williamson 2000, 200; Maher 2004, sec. 3.2; Conee and Feldman 2010, 129; and Kvanvig 2010, sec. 2.

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than *G. balachowskyi*. Meanwhile, several disgruntled lab assistants have started sending out the very same sort of letter along with specimens from the latter culture. On receiving your letter, which happens to be from the diligent lab assistant, you assign more credence to the proposition that your specimens are *G. hackmani* than to the proposition that they are *G. balachowskyi*.

Sue and her friend Bob enter a psychology study. The study proceeds as follows: each woman is given questionnaires that indicate whether she finds her friend attractive. If she does, she is injected with an anxiety-producing drug before meeting her friend. If she does not, she is injected with a saline solution. Sue is not told about the nature of the experiment. She does probably like Bob as more than a friend. On receiving the anxiety-producing drug and meeting Bob, Sue reflects on her fluttering nerves and raises her credence that she likes Bob as more than a friend.⁹

In both cases, agents rationally update their credences on a constraint delivered by their experience. In the entomology case, your relative confidence that your specimens are *G. hackmani* is justified on the basis of the letter you receive. In the psychology case, Sue's confidence that she likes Bob is justified on the basis of her fluttering nerves. And in both cases, agents arrive at just the right credences to have. In the first case, your specimens are indeed probably *G. hackmani*. And in the second, Sue does indeed probably like Bob as more than a friend.

And yet: in both cases, there is something epistemically incorrect about the agents' credal states. As a symptom of this incorrectness, notice that the following variants of (19) and (20) are intuitively false:

- (22) On receiving your flies from the entomology supply company, you know that it is more likely that they belong to *G. hackmani* than to *G. balachowskyi*.
- (23) Sue knows that she probably likes Bob as more than a friend.

The entomology case resembles the fake barn case by Carl Ginet, cited in Goldman 1976. The psychology case resembles the traditional Gettier cases in Gettier 1963. Both instances of resemblance raise our second problem, namely how to give a uniform account of the epistemic incorrectness of Ginet beliefs and the credences in (22), and Gettier beliefs and the credences in (23). As Pritchard (2005) would put it, the former result from "environmental luck" and the latter result from "intervening

9. See Dutton and Aron 1974 for a similar experiment demonstrating the misattribution of arousal by study participants.

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luck.” A less-than-satisfying response would be to say that the absence of luckiness is simply a primitive epistemic virtue that properties of credence distributions may exhibit. A more satisfying theory would identify positive epistemic properties that properties of credence distributions may share with beliefs, and explain why these properties are not instantiated in the above examples.

The above Gettier cases were constructed to accompany naturally occurring probabilistic knowledge ascriptions. Standard Gettier cases provide a host of further examples of Gettiered partial beliefs. Consider Henry from Goldman 1976. While driving through fake barn country, Henry does not know that it is more likely than not that he is looking at a barn. Since Henry is indeed looking at a barn, it is more likely than not that he is looking at a barn. And his high credence that he is looking at a barn may be justified. But intuitively his high credence does not constitute knowledge. Similarly, consider Smith from Gettier 1963. Smith intuitively does not know that the man who will get the job probably has ten coins in his pocket, even though the man probably does have ten coins in his pocket and Smith has a justified belief that he probably does. And Smith does not know that it is very likely that either Jones owns a Ford or Brown is in Barcelona, even though that is very likely and Smith has a justified belief that it is very likely. We can repeat this reasoning for any Gettier case where the very same evidence justifies an agent’s full belief and high credence in a proposition. Both the full belief and the high credence will intuitively fail to constitute knowledge in such cases. However, the examples I have introduced helpfully demonstrate that probabilistic Gettier cases need not have this structure. In the entomology case, you may not even have had a full belief that your flies were *G. hackmani*. In the psychology case, Sue may not even have had a full belief that she liked Bob as more than a friend. These cases highlight that failures of probabilistic knowledge are not always immediately parasitic on failures of propositional knowledge.

A natural conservative response to the above examples is to try to give a less immediate explanation of the epistemic incorrectness of the relevant credences in terms of the epistemic incorrectness of full beliefs. For instance, one might say that the incorrect credences in each case are *grounded in* full beliefs that fail to constitute knowledge. I discuss similar strategies in detail in section 5.2, but it may be useful to note in advance that the conservative response faces several challenges. For instance, the most natural way of spelling out the claim that your credences are grounded in certain propositional beliefs is to say that your credences are the

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result of your conditionalizing on certain propositions. But this claim conflicts with our initial rejection of strict conditionalization as an acceptable updating procedure. Furthermore, the conservative response forecloses on the possibility of a certain sort of “Probability First” epistemology, according to which all epistemic facts about agents can ultimately be understood in terms of facts about their credence distributions and value functions.¹⁰ And even for opponents of Probability First theories, it is difficult to point out the full beliefs that allegedly ground the incorrect credences in the above examples. The fact that Sue gives a certain amount of credence to the proposition that she likes Bob may not even supervene on facts about her full beliefs.

3. A Solution: Probabilistic Knowledge

The problems raised in sections 1 and 2 are obviously intertwined. For instance, in the section 2 cases, one ascribes epistemically incorrect credences using belief reports embedding language of subjective uncertainty, and the corresponding knowledge reports are infelicitous. Also, if we adopt an expressivist semantics for assertions in order to address the problems raised in section 1, then assertions such as (21) constitute evidence that directly constrains our credences:

- (21) If you are in Red Army territory, you are .75 likely in their Headquarters Company Area.

And that means that one can easily manufacture more cases like those in section 2, since it is easy to manufacture deviant circumstances under which testimony imparts justification but not knowledge.

At this point, the most straightforward solution to both problems is relatively conspicuous: *properties of your credence distribution can constitute knowledge*. In other words: it is commonly said that some beliefs amount to knowledge, or count as knowledge, or constitute knowledge. In just this same sense, partial beliefs (or: *credences*) can constitute knowledge.¹¹ The

10. The ‘Probability First’ handle is due to Weatherston 2005.

11. I follow many authors in using ‘constitutes’ for the relevant relationship between your doxastic and epistemic states. To cite a few examples: Alston (1988, 270) mentions the requirement that a “grounding belief constitute knowledge”; Plantinga (1996, 309) says that in a Gettier case, “your belief is justified and true, but doesn’t constitute knowledge”; Williamson (2004, 284) claims that “a flat-out belief is fully justified if and only if it constitutes knowledge”; and Fumerton (2006, 26) discusses whether “an apparently jus-

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same goes for conditional credences and, more generally, for arbitrary properties of your credence distribution.

Given this simple thesis, we can accept the self-evident claim that ascriptions such as (22) ascribe knowledge, while also accepting the compelling evidence that (22) does not ascribe an attitude toward truths.

- (22) You know that it is more likely that your specimens belong to *G. hackmani* than to *G. balachowskyi*.

We can also give an attractively simple theory of the cases discussed in section 2. The relevant properties of credence distributions are epistemically incorrect because they fail to constitute knowledge. They are exactly like traditional Gettier beliefs in this respect, and that is what unifies the section 2 cases with more standard cases where an agent has a justified true belief without having knowledge.

One could argue for the necessity of probabilistic knowledge by canvassing alternative answers to the problems in sections 1 and 2 and arguing that none of them is viable. I briefly discuss some alternative answers in section 5.2. But my main aim in this paper is more modest: to build a positive case for a theory of probabilistic knowledge. A positive case for a theory has two parts: expounding the virtues of the theory and tempering its flaws. A virtue of probabilistic knowledge is that it yields simple solutions to problems that I discuss in sections 1, 2, and 6. An apparently serious flaw of probabilistic knowledge is that it seems to fly in the face of our intuition that knowledge is a factive attitude, as well as traditional epistemological claims that take that intuition for granted. In the next section, I argue that this flaw is not as serious as it first appears.

4. Factivity, Safety, and Sensitivity

Let us examine the argument against probabilistic knowledge in more detail. It is widely agreed that knowledge is a factive mental state and that ‘knows’ is a factive attitude verb.¹² In fact, a natural thought is that knowledge ascriptions are valuable in part because knowledge is factive, and ascriptions of knowledge therefore communicate substantive information about something other than their subject. By learning that John

tified true belief *constitutes* knowledge.” Like these authors, I use ‘constitutes’ for a metaphysically innocuous relation.

12. See Stjernberg 2009 and Hazlett 2010 for some expressions of dissent. Their objections are orthogonal to the present project.

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knows that it is raining, you learn that it is raining. By saying that John knows that it is raining, I can tell you that it is raining.

A number of theorists implicitly rely on the following schema in order to say what it is for 'knows' to be factive, where 'S' is to be replaced by the name of an agent and '*p*' by a declarative sentence:

(FACTIVE₁) For any world *w*, if 'S knows that *p*' is true as evaluated at *w*, then '*p*' is true as evaluated at *w*.

If the semantic value of '*p*' is a constraint on credence distributions, then the conclusion '*p*' is not truth-apt. And so the opponent of probabilistic knowledge might infer that instances of (FACTIVE₁) are generally false when '*p*' is replaced by language of subjective uncertainty. The opponent of probabilistic knowledge concludes: agents cannot bear the knowledge relation to properties of credence distributions.

The advocate of probabilistic knowledge has a ready response. Our ordinary use of knowledge ascriptions does not constitute evidence for (FACTIVE₁) as opposed to the following underspecified claim:

(FACTIVE₂) The inference from 'S knows that *p*' to '*p*' is valid.

If we restrict our attention to instances of (FACTIVE₂) where the semantic value of '*p*' is a proposition, then (FACTIVE₂) should be equivalent to (FACTIVE₁). But a more general notion of validity is required in order to capture our intuitions about instances of (FACTIVE₂) where the semantic value of '*p*' is a constraint on credences. For example, suppose I tell you the following:

(24) John knows that it is more likely that your specimens belong to *G. hackmani* than to *G. balachowskyi*.

Just as in the propositional case, this ascription communicates information about subjects other than John. By learning that John knows that it is more likely that your specimens are *G. hackmani* than that they are *G. balachowskyi*, you learn that it is more likely that your specimens are *G. hackmani*. By uttering (24), I communicate that it is more likely that your specimens are *G. hackmani*. In other words, according to the expressivist, I advise you to give more credence to the proposition that your specimens are *G. hackmani* than to the proposition that they are *G. balachowskyi*. If I do not endorse this advice, it is inappropriate for me to utter (24).

This example points to an appropriate interpretation of (FACTIVE₂) for expressivists. The expressivist need not say that a factive

ascription is true only if its prejacent is true, but may instead say that ‘*S* knows that *p*’ entails ‘*p*’ in the following sense: if you follow the advice that ‘*S* knows that *p*’ expresses, you thereby follow the advice that ‘*p*’ expresses. In other words: every credal state that satisfies the constraint expressed by a factive ascription also satisfies the constraint expressed by its prejacent. For example: if the semantic value of a sentence is a set of probability measures, as on the naïve expressivist theory outlined in section 1, then a factive ascription entails its prejacent because the semantic value of the former is a subset of the semantic value of the latter.

This liberal understanding of factivity is not necessarily at odds with (FACTIVE₁). Given a carefully chosen theory of truth, an expressivist may ultimately accept both characterizations of factivity stated above. For instance, an expressivist may say that uttering a factive construction commits you to the *truth* of its complement, where this commitment merely involves a commitment to the *complement itself*. In particular, an expressivist about epistemic modals may endorse a theory of truth according to which “to believe that *S* is true is just to agree with *S*, and to believe that *S* is false is just to disagree with *S*” (Schroeder, n.d., 10), and then characterize factivity by saying that you should utter a factive construction only if you agree with its complement. Gibbard (2003) and Schroeder (2008) develop accounts of truth on behalf of expressivists about ethical discourse, and their accounts are similarly hospitable to (FACTIVE₁).¹³ The present point is merely that (FACTIVE₁) is not essential to our notion of factivity. The main requirement on a probabilistic definition of factivity is that it should yield the standard definition of factivity as a special case, so that the probabilistic definition still applies when restricted to factive operators not embedding language of subjective uncertainty.

Using a suitably general interpretation of (FACTIVE₂) to define factivity lets us predict judgments about when we can utter probabilistic knowledge ascriptions. For instance: the expressivist should accept some norms of assertion, such as the norm that you should not utter a factive ascription unless you endorse the advice that it expresses. On the most straightforward expressivist account of endorsing advice, that just means that your credal state must itself be a member of the semantic value of the ascription. From (FACTIVE₂), it follows that your credal state is a member of the semantic value of the prejacent. Hence you should not utter a

13. For further discussion, see Schroeder 2008, secs. 11.4–6. I shall continue presupposing a naïve expressivist semantics for ease of exposition, though the following discussion could be tailored to accommodate a variety of more nuanced expressivist theories.

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factive ascription unless you endorse the advice expressed by its prejacent. And this conclusion is borne out by our judgments about factive ascriptions, as noted with (24) above.

Using (FACTIVE₂) to define factivity also helps us explain why certain constraints cannot be embedded in factive ascriptions. For example, note that (25) is infelicitous, while (26) sounds fine:

- (25) #John knows that it is probably raining, and Bob knows that it probably isn't.
- (26) John thinks that it is probably raining, and Bob thinks that it probably isn't.

Since knowledge is factive, (25) entails 'it is probably raining' and 'it probably isn't raining'. The semantic values of these sentences are incompatible constraints. Hence (25) gives advice that is inconsistent, just like the advice to believe both that it is raining and that it is not raining. A similar but more involved argument predicts that the following utterance is infelicitous:

- (27) #John knows that it's very probably raining, and Mary knows that if it is more likely than not raining, then it is probably Sunday. But it is Tuesday.

In order to derive this result, it is necessary to have some semantics in place for indicatives with constraints as antecedents. In the spirit of Yalcin 2007, let us suppose that the semantic value of an indicative contains a probability measure just in case the result of updating that measure on the antecedent constraint satisfies the consequent constraint. First, note that any measure that satisfies the semantic value of 'it's very probably raining' satisfies the semantic value of 'it is more likely than not raining'. Second, note that updating a measure on a constraint that the measure already satisfies should yield the measure itself as a result.¹⁴ It follows that any measure that satisfies the semantic value of 'it's very probably raining' and 'if it is more likely than not raining, then it is probably Sunday' should satisfy the semantic value of 'it is probably Sunday'. And since knowledge is factive, it follows that any measure that satisfies the semantic value of the first sentence of (27) should satisfy the

14. It is difficult to give a procedure for updating an arbitrary probability measure on an arbitrary constraint, for reasons mentioned in section 3. But it should be relatively uncontroversial that the stated result will be a feature of any reasonable updating procedure.

semantic value of ‘it is probably Sunday’. But arguably one cannot believe both that it is probably Sunday and that it is Tuesday. Whether or not you endorse the Lockean thesis, it seems plausible that there are some rational connections between full and partial belief states, and that among them is the claim that one cannot believe a proposition without giving it more credence than its negation. And so the first and second sentences of (27) express inconsistent advice. To sum up: any expressivist must develop notions of consequence, validity, and inconsistency within the context of an expressivist semantic theory. And using these notions, the expressivist can account for a range of ordinary language judgments about knowledge ascriptions by endorsing the claim that one can validly infer from a knowledge ascription to its complement.

One might object that factivity should be defined using (FACTIVE₁) rather than (FACTIVE₂), since *factive* verbs should relate their subjects to *facts*. Ultimately, I am not interested in settling a terminological dispute over the most appropriate or natural definition of ‘factive’. (FACTIVE₁) is already an idiosyncratic definition compared with the standard use of ‘factive’ in the linguistics literature. Following an early discussion in Kiparsky and Kiparsky 1970, semanticists distinguish *factive* from *entailing* verbs. A verb is *factive* just in case its content-clause complement is normally presupposed. A verb is *entailing* just in case its complement is entailed by positive declarative sentences containing the verb.¹⁵ In this terminology, the epistemologically important feature of knowledge characterized by (FACTIVE₁) is that it is an *entailing* mental state. Whatever this epistemologically important feature is called, my claim is that (FACTIVE₂) effectively captures our intuitions about the feature, and thereby lets us predict many ordinary language judgments about probabilistic knowledge.

In light of the cases given in section 2, some theorists may be pessimistic about analyses of propositional knowledge. But even without attempting an analysis of probabilistic knowledge, one can investigate the qualities that make it valuable and distinguish it from other mental states. Factivity is one example. Safety is another. Safe attitudes are valuable

15. For further discussion of *factive* and *entailing* verbs, see Huddleston and Pullum 2002, sec. 7.4. In addition to these notions, one might introduce a further notion of factivity, stated as follows: necessarily, if *S* knows that *p*, then *p*. As elaborated above, (FACTIVE₂) corresponds to a notion of validity roughly analogous to that introduced in Kaplan 1989, whereas this further notion of factivity says that inferences from knowledge ascriptions to their prejacent are modally valid.

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because they preclude a certain sort of epistemic fragility exhibited by the justified credences in the section 2 cases. And just as with factivity, our traditional notion of safety naturally extends to probabilistic knowledge. Consider the following simple statement of the safety condition from Williamson (2000, 128):

(SAFE₁) For all cases α and β , if β is close to α and in α one knows that C obtains, then in β one does not falsely believe that C obtains.

It is not difficult to reformulate (SAFE₁) so that it applies to probabilistic knowledge:

(SAFE₂) For all cases α and β , if β is close to α and in α one knows that C , then the following is not the case in β : that one believes that C , and it is not the case that C .

Instances of (SAFE₁) result from replacing ‘ C ’ with an expression that refers to a proposition. Instances of (SAFE₂) result from replacing ‘ C ’ by a sentence. That sentence may contain language of subjective uncertainty. If it does, the resulting instance of (SAFE₂) will contain expressions of subjective uncertainty embedded under four operators: the universal quantifier, conditional, and intensional operator ‘in β ’, as well as ‘believes’ or negation. Any complete semantics for expressions of subjective uncertainty should settle how they interact with these operators. But even without using a semantic theory to generate truth conditions for instances of (SAFE₂), we can rely on ordinary language intuitions in assessing its instances in particular cases. And just as with (FACTIVE₂), instances of (SAFE₂) let us predict judgments about probabilistic knowledge ascriptions. For instance, recall that the following ascriptions are felicitous in some cases:

- (19) I hereby let you know that it is more likely that your specimens belong to *G. hackmani* than to *G. balachowskyi*.
- (20) I realized that I probably liked that guy as more than a friend.

But in the deviant entomology and psychology cases in section 2, the following variants of (19) and (20) are intuitively false:

- (22) You know that it is more likely that your specimens belong to *G. hackmani* than to *G. balachowskyi*.
- (23) Sue knows that she probably likes Bob as more than a friend.

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The safety condition (SAFE₂) accounts for our judgments that (22) and (23) are false. In the entomology case, you might easily have received your specimens from a disgruntled lab assistant instead of the diligent assistant. And so it might easily have been that your specimens were not more likely *G. hackmani* than *G. balachowskyi*, but you still believed that they were more likely *G. hackmani* on the basis of testimony from the entomology supply company. In the psychology case, it might easily have been that the psychology researchers injected subjects with anxiety-producing drugs just in case they were not attracted to their friends. And so it might easily have been that Sue in fact probably didn't like Bob, but still believed that she probably liked him on the basis of her fluttering nerves.

Finally, notions of sensitivity also extend to probabilistic knowledge. For instance, Nozick 1981 states the following sensitivity condition on knowledge:

(SENSITIVE₁) *S* knows, via method (or way of believing) *M*, that *p* only if: if *p* weren't true and *S* were to use *M* to arrive at a belief whether (or not) *p*, then *S* wouldn't believe, via *M*, that *p*.

The advocate of probabilistic knowledge may endorse the following deflationist variant of (SENSITIVE₁):

(SENSITIVE₂) *S* knows, via method (or way of believing) *M*, that *p* only if: if it were not the case that *p* and *S* were to use *M* to arrive at a belief whether (or not) *p*, then *S* wouldn't believe, via *M*, that *p*.

Here again, the expressivist may capture the spirit of a statement concerning the truth conditions of ordinary propositions by endorsing a principle in which schematic letters are to be replaced by sentences containing expressions of subjective uncertainty.¹⁶

The deflationist strategy applied repeatedly in this section is not open just to expressivists. Advocates of dynamic and relativist semantic theories may endorse similar deflationist theories of factivity, safety, and sensitivity. In just this sense, my defense of probabilistic knowledge has been neutral between various semantic theories of language of subjective uncertainty.¹⁷ However, the deflationist strategy does serve to remind us of the fact that it is no small feat to give a complete semantics for ex-

16. As noted above, some expressivists may endorse a deflationist notion of truth and thereby automatically reinterpret (SAFE₁) as (SAFE₂), (SENSITIVE₁) as (SENSITIVE₂), and so on, for similar pairs of principles.

17. For further discussion of factive constructions in dynamic frameworks, see

pressions of subjective uncertainty. In particular, such expressions commonly occur not only in belief reports, but also under overt intensional operators (as in (SAFE₂)), and in the antecedents of subjunctives (as in (SENSITIVE₂)). Furthermore, I have considered only a small handful of definitions of factivity, safety, and sensitivity; other definitions may contain other intensional operators. It is not at all obvious what an intensional semantics for non-truth-conditional expressions should look like; on a traditional semantics, intensional operators are just those operators that shift the world at which their argument is evaluated for truth.

At this juncture, some might take intensional contexts as an insuperable difficulty for non-truth-conditional analyses of the language of subjective uncertainty, preferring instead to wrestle with the host of arguments against truth-conditional analyses canvassed in section 1. I do not aim to refute that position here, but simply to set out a strong positive case for probabilistic knowledge. For the many theorists that endorse non-truth-conditional theories of the language of subjective uncertainty, it is good news that the core features of factivity, safety, and sensitivity can be applied to probabilistic mental states.

5. Several Decision Points for Advocates of Probabilistic Knowledge

The foregoing discussion raises several questions for advocates of probabilistic knowledge. Can other properties of mental states constitute knowledge, aside from properties of your credence distribution? Can probabilistic knowledge be analyzed in terms of propositional knowledge or other familiar epistemic notions? How does probabilistic knowledge relate to recent themes in formal epistemology? Do ascriptions of probabilistic knowledge have truth conditions? And finally, what less radical morals can be taken from this discussion by those who resolutely endorse the claim that the objects of knowledge must have truth conditions? In this section, I discuss each of these questions in turn.

5.1. *Nonprobabilistic Nonpropositional Knowledge*

Some expressions of subjective uncertainty are not easily modeled by the naïve expressivist theory outlined in section 1. The less tractable expressions include three of our original examples:

Karttunen 1974 and Beaver 2001, chap. 6. For the development of a disquotational notion of truth in a relativist framework, see Egan, Hawthorne, and Weatherson 2005.

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- (1) John might be in his office.
- (5) John must have gone home early today.
- (6) If John is still in the building, he is in his office.

For the naïve expressivist, the most natural account of these expressions appeals to thresholds. (1) advises you to give at least a certain small amount of credence to the proposition that John is in his office. (5) advises you to give at least a certain large amount of credence to the proposition that John went home early. (6) advises you to give at least a certain large amount of credence to the proposition that John is in his office, conditional on his still being in the building. But unfortunately, unless the threshold credence in question is 0 for ‘might’ and 1 for ‘must’ and conditionals, these proposals fail to validate intuitive inference rules, such as:

- (28) From ‘might (p or q)’, infer: ‘might p or might q ’.
- (29) From ‘must p ’ and ‘must q ’, infer: ‘must (p and q)’.
- (30) From ‘if p , q ’ and ‘if p , r ’, infer: ‘if p , (q and r)’.¹⁸

And it would be a mistake for the naïve expressivist to accept extreme thresholds for these expressions. To do so would conflate epistemic possibility with nonzero probability.¹⁹ This problem for the naïve expressivist has a familiar shape. In deciding what minimal conditional credence one ought to have in order to count as having updated on an indicative conditional, one wants to say: credence sufficient for conditional belief. But identifying such a threshold engenders familiar complications for the Lockean project of analyzing belief in terms of sufficient credence: belief does not entail certainty, for instance, so the Lockean must deny closure for belief.

A better expressivist account of (1), (5), and (6) requires semantic resources other than sets of probability measures. For instance, expressivists might use sets of possibilities as part of their representation of agents’ doxastic states, à la Hintikka 1962. They could then say that modals and conditionals are used to express advice about that set of doxastic possibilities: (1) ensures that it contains some worlds where John is in his office, (5) ensures that it contains only worlds where John went home

18. It is generally assumed that (28) is valid; for further discussion, see Kamp 1974, Zimmermann 2000, Simons 2005, and Geurts 2005. (29) is a theorem of **K** (see Hughes and Cresswell 1968, 27).

19. See Yalcin 2007, 1016 for a related discussion and McGee 1994 and Hájek 2003 for corroborating arguments.

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early, and (6) ensures that if it contains a world where John is still in the building, that world is one where John is still in the building. Another expressivist strategy for indicatives simply accepts conditional belief as a primitive mental state and says that (6) expresses advice regarding instances of that state. Either of these strategies will prompt a nonprobabilistic account of knowledge ascribed by sentences such as:

- (31) Bob knows that John might be in his office.
- (32) Bob knows that John must have gone home early today.
- (33) Bob knows that if John is still in the building, he is in his office.

In other words, there may be *nonpropositional* knowledge other than *probabilistic* knowledge. Fortunately, many arguments in sections 2–4 extend easily to nonprobabilistic nonpropositional knowledge. For instance, an expressivist may appeal to factivity to explain why (34) is infelicitous:

- (34) #John knows that it can't be raining, and Bob knows that it might be.

According to the expressivist, (34) entails inconsistent advice, namely that your doxastic possibilities contain no worlds where it is raining, and that they contain some worlds where it is raining.

Judging the bounds of nonprobabilistic nonpropositional knowledge is a vast project. Expressivists about ethical discourse have developed quasi-realist accounts of moral vocabulary in knowledge ascriptions and other factive environments.²⁰ It is just as important to develop accounts of factive environments if one rejects truth-conditional semantic theories of predicates of personal taste, future contingents, or aesthetic discourse. The scope of this paper is limited to the case for probabilistic knowledge, which is particularly well supported by contemporary theories of the semantics of subjective uncertainty and the epistemology of updating, as well as the arguments given in section 6.

5.2. Analyzing Probabilistic Knowledge

It is worth investigating whether probabilistic knowledge can ultimately be analyzed or replaced by more familiar epistemic notions. For instance, traditional epistemologists frequently discuss the *epistemic probability* of

20. For discussion, see Blackburn 1996 and 1998, Gibbard 2003, Ridge 2007, Schroeder 2008, and Chrisman 2010.

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propositions. For some epistemologists, epistemic probability is by definition closely tied with propositional knowledge. It is natural to wonder whether epistemic probability might yield conventional solutions to problems that motivated the introduction of probabilistic knowledge. Responsibility dictates that we dismiss conventional proposals before embracing more revisionary ones. Here is one natural proposal: a property of your credence distribution constitutes knowledge just in case your epistemic probability function has that property. For instance, your credence in a proposition constitutes knowledge just in case it equals your epistemic probability for that proposition.

In order to evaluate this proposal, it is necessary to have a clear definition of ‘epistemic probability’ at hand. There is a family resemblance among the numerous definitions of ‘epistemic probability’ in the literature. A proposition having high epistemic probability for you is generally associated with your having justification or evidence for believing that proposition:

It is a truism that a belief is justified if and only if its epistemic probability is sufficiently high. (Pollock and Cruz 1999, 110)

This is what we shall mean by ‘evidential probability’: the evidential probability of a statement S , relative to a body of knowledge Γ , is the interval $[p, q]$ determined by the unsharpenable *evidence* Δ contained in Γ bearing on S . (Kyburg and Teng 2001, 219)

Another common thread is that your epistemic probability for a proposition is the degree of belief in that proposition that it is rational for you to have:

Roughly, the epistemic probability of a proposition can be thought of as the degree of credence—that is, degree of confidence or belief—we rationally should have in the proposition. Put differently, epistemic probability is a measure of our rational degree of belief under a condition of ignorance concerning whether a proposition is true or false. (Collins 1999, 74)

Relative to K , p is epistemically more probable than q , where K is an epistemic situation and p and q are propositions, just in case any fully rational person in K would have a higher degree of belief in p than in q . (Draper 1989, 349)

In the same vein, van Inwagen (1996, 221) says that the epistemic probability of a proposition relative to an epistemic situation equals the odds

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that a “fully rational ideal bookmaker” would give to the proposition if the bookmaker were in that epistemic situation.

These definitions point to a central difficulty for saying that properties of your credences are knowledge just in case they are properties of your epistemic probabilities. For many accounts of justification, evidence, or rational belief, the identification returns the wrong verdict in probabilistic analogues of Gettier cases. Subjects in Gettier cases have justification and evidence for their beliefs, and their beliefs constitute rational responses to that evidence. This claim is not only entailed by a number of theories of justification but also demanded by our intuitions about the cases. For example: in the psychology case, Sue intuitively has evidence for the proposition that she likes Bob as more than a friend, namely that her nerves start fluttering as soon as she sees him, and so Sue has a high epistemic probability for the proposition that she likes Bob. But her high credence in the proposition that she likes Bob does not constitute knowledge. Hence it is not the case that properties of your credence distribution constitute knowledge just in case they are properties of your epistemic probability distribution. This problem for the proposed analysis of probabilistic knowledge does not depend on an internalist conception of evidence. For instance: suppose (à la Williamson 2000) that your evidential probability function is the result of conditionalizing a distinguished initial credence distribution on all and only the propositions that you know. In the psychology case, Sue knows that her nerves start fluttering as soon as she sees Bob. So even on this account, her evidential probability that she likes Bob may match her high credence, without that high credence constituting knowledge.

There are several further problems for an objective Bayesian definition of probabilistic knowledge in terms of evidential probability, according to which you know exactly those properties of a distinguished initial credence distribution conditionalized on your evidence. Problems arise even if we adopt an externalist theory of evidence according to which you do not have evidence for your true beliefs in Gettier cases. First, it is not clear that any facts exist that could entirely determine the identity of the distinguished initial distribution. Second, the proposal does not generalize easily to properties of other mental states that could constitute knowledge, such as conditional beliefs (see section 5.1). Third, the proposal contradicts the intuitive claim that your high credence in some evidence proposition may constitute probabilistic knowledge. For example: suppose that Sue knows that it is .99 likely that her nerves started fluttering. The proposal entails that her epistemic

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probability for the proposition that her nerves started fluttering is merely .99. Hence the proposition cannot be evidence for Sue. But intuitively, such propositions can indeed constitute evidence for Sue. Fourth, it is not obvious that your actual credences ever exactly match the result of conditionalizing a distinguished credence distribution on your evidence, especially given that the initial distribution is notoriously difficult to define. But on the present proposal, that means that it is not obvious that your actual credences ever constitute knowledge.

The objective Bayesian could address the last two concerns by introducing another proposal, namely, that a property of your credence distribution constitutes knowledge just in case it closely enough resembles some property of your epistemic probability function. But this second proposal precludes plausible closure principles for probabilistic knowledge. For example, the proposal suggests that one may know that it is probably snowing, and that it is not July if it is probably snowing, while failing to know that it is not July. Closure may be recovered by yet another proposal, namely that a property of your credence distribution constitutes knowledge just in case it is a property of the distribution that best estimates your epistemic probability function, given your credences about your epistemic probabilities. But this third proposal yields counter-intuitive verdicts. For instance, the simple knowledge that it is probably raining outside is intuitively available even to subjects that are incapable of estimating epistemic probabilities. And finally, a general problem arises for nearly any analysis of probabilistic knowledge in terms of epistemic probability. On many accounts of epistemic probability, your credence in a proposition might equal your epistemic probability for that proposition simply as a matter of coincidence. And in such cases, it is not clear that the relevant property of your credence distribution should thereby constitute knowledge.

As a final experiment, we may consider “higher-order” accounts according to which you know that it is probably raining just in case it is likely given your evidence that it is *probably* raining.²¹ Unfortunately, these accounts fail to identify a sufficient condition for probabilistic knowledge. For example, suppose a health official consults many diverse panels of experts about whether smoking is hazardous to your health, and every expert says that smoking is probably not hazardous. Then intuitively speaking, given the official’s evidence, it is likely that smoking is probably

21. I am grateful to an anonymous referee for suggesting that I consider this proposal.

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not hazardous. But the official does not thereby know that smoking is probably not hazardous, at least in part because smoking probably is hazardous. To take another example: it may well be that in the entomology case, it is not only likely on your evidence that your flies are *G. hackmani*, but also likely on your evidence that they are probably *G. hackmani*. For instance, suppose your letter about your flies says that they are probably *G. hackmani*, and your evidence suggests that your letter is almost certainly trustworthy. Then intuitively speaking, given your evidence, it is almost certainly the case that your flies are probably *G. hackmani*. But you do not thereby know that your specimens are probably *G. hackmani*. To sum up both examples: even partial beliefs made likely by higher-order evidence may be incorrect or subject to Gettier cases, and thereby fail to constitute knowledge.

5.3. Probabilistic Knowledge and Recent Themes in Formal Epistemology

How are probabilistic knowledge norms related to more conventional norms governing credences?²² In answering this question, it is useful to reflect on the multiplicity of norms governing full beliefs. In some sense, you ought to maximize the truth of your full beliefs. In another sense, you ought to have full beliefs that constitute knowledge. Both claims are intuitive. The simplest response to this pair of intuitive judgments is pluralism: full belief is governed by a variety of norms. Just the same goes for norms governing partial beliefs. In some sense, you ought to maximize the expected accuracy of your credences. In another sense, you ought to have credences that constitute probabilistic knowledge.

These various norms governing full and partial belief are related in interesting ways. The debate over peer disagreement presents a helpful case study in applying knowledge-based norms to extant formal epistemology literature. A natural suggestion is that any compromise of peer opinions ought to preserve properties of those opinions that constitute knowledge. This suggestion does not deliver a global constraint on legitimate judgment aggregation procedures.²³ But knowledge preservation

22. I am grateful to an anonymous referee for prompting me to discuss each topic covered in this section.

23. In some cases, unanimous judgments of probabilistic independence will constitute knowledge for each peer. In some cases, unanimous judgments of probability will constitute knowledge. And no aggregation procedure can always preserve both (see French 1985). Hence no aggregation procedure always preserves properties of credence distributions that constitute knowledge.

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may still constrain any instance of judgment aggregation. Say that Alice and Bob are peers with different credence distributions. Any aggregation procedure will determine an ordering of candidate credence distributions on which Alice and Bob might compromise.²⁴ A knowledge-based norm on aggregation would dictate that Alice and Bob should compromise by adopting the best credence distribution that has all the properties of their credences that constitute knowledge. There are several ways of spelling out this norm in detail. For instance, perhaps compromise must preserve only common knowledge, or perhaps mutual knowledge or distributed knowledge should be preserved. However we answer these questions, probabilistic knowledge will play an important role in spelling out a knowledge-based norm governing how Alice and Bob should compromise.

The Principal Principle introduces a helpful case study in the opposite direction, namely how we can apply existing formal epistemology literature to derive knowledge-based norms for credences. Suppose that you know that your friend Elmer is an expert at predicting earthquakes. In other words, you know that Elmer believes that an earthquake will probably happen if and only if an earthquake will probably happen. Then if you know what Elmer believes, you will also know whether an earthquake will probably happen. Hall (1994) and Van Fraassen (1989) argue that the objective chance function is like an expert who gives you testimony about the likelihood of various events. Their arguments suggest the following analogy. Suppose that you know the following cousin of the Principal Principle: an earthquake has high objective chance if and only if it will probably happen.²⁵ If you know the objective chance of an earthquake, then you will also know whether it will probably happen. As long as knowledge is closed under known entailment and you know the appropriate cousin of the Principal Principle, you may gain probabilistic knowledge from your propositional knowledge of objective chances.

Again, it is important to avoid overstating the importance of probabilistic knowledge norms. They are not intended as replacements for familiar Bayesian norms. In just the same sense as before, you should

24. For example: the procedure of “splitting the difference” determines that the arithmetic mean of their credence distributions constitutes the best compromise of their opinions, and we may order other credence distributions by distance from that perfect compromise, according to some metric on measures.

25. This biconditional about earthquakes does not explicitly concern rational credences. But accepting the biconditional has the effect of making your credences satisfy a variant of the Principal Principle.

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maximize the expected accuracy of your credences and make your credences conform to the Principal Principle. These norms do not always recommend the same credences as probabilistic knowledge norms. But both spell out intuitive and important respects in which we value various partial beliefs, just as familiar knowledge and accuracy norms spell out diverse respects in which we value various full beliefs.

5.4. *The Semantics of Probabilistic Knowledge Ascriptions*

Suppose that your friend Lucy is perfectly aware that her odds of winning the lottery are horrible, but she buys a ticket anyway. Suppose that neither of you has read the newspaper to see if she won the lottery. Before you read the paper, it seems perfectly fine for you to say that Lucy knows that she probably lost the lottery. But suppose that later you read the paper and see that Lucy actually has the winning ticket number. Now you will no longer ascribe the same probabilistic knowledge to Lucy. Now you will say that Lucy does not know that she probably lost the lottery, because she *didn't* probably lose; she won. And that may seem a bit puzzling. After all, how could some fact about Lucy have changed simply because you read the newspaper?

The above puzzle involves ascribing probabilistic knowledge to Lucy. But in fact, this sort of puzzle does not fundamentally concern knowledge or knowledge ascriptions. Consider the following close variant of the puzzle: suppose that your friend Larry is perfectly aware that his odds of winning the lottery are horrible, but he buys a ticket just for fun. Suppose that neither of you has read the newspaper to see if he won the lottery. Before you read the paper, it seems perfectly fine for you to say that Larry probably lost the lottery. But suppose that later you read the paper and see that Larry actually has the winning ticket number. Now you will say that Larry *didn't* probably lose the lottery; he won. And that may seem a bit puzzling. After all, how could some fact about Larry have changed simply because you read the newspaper?

These similar puzzles call for similar answers. Looking at the newspaper does not have some mysterious impact on facts about Lucy or Larry. In both cases, the puzzle arises only if we mistakenly assume that a certain sentence you later come to reject was reporting some fact to begin with. Instead we should say that both 'Lucy knows that she probably lost' and 'Larry probably lost' demand an alternative semantics according to which they do not report facts at all. This answer is more obvious for the second puzzle above, but just as viable for the first. Any semantics for language of

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subjective uncertainty will answer detailed questions about how to evaluate ‘Larry probably lost’ from various perspectives represented in the case above. Advocates of probabilistic knowledge should answer questions about ‘Lucy knows that she probably lost’ in just the same way. For example: you should accept that up until you read the paper, you believed that Larry probably lost the lottery. And you should accept that up until you read the paper, you believed that Lucy knew she probably lost the lottery. But you should deny that up until you read the paper, Larry did probably lose the lottery. And you should deny that up until you read the paper, Lucy did know that she probably lost the lottery.

Some may find it hard to accept that some knowledge ascriptions lack straightforward truth conditions. But this claim is much less controversial than it first appears. Often theorists who adopt an expressivist account of some vocabulary also adopt an expressivist account of factive constructions embedding that vocabulary. The literature on ethical expressivism is a case in point. If you deny that ‘murder is wrong’ has straightforward truth conditions, then denying the same for ‘Martha knows that murder is wrong’ is par for the course. Expressivists about language of subjective uncertainty should recognize that probabilistic knowledge ascriptions have both factual and nonfactual entailments. For example, (35) entails both factual sentences such as (36) and non-factual sentences such as (37):

- (35) Nelson knows that it is probably raining.
- (36) Nelson believes that it is probably raining.
- (37) It is probably raining.

For expressivists, probabilistic knowledge ascriptions are by no means special in this respect. For example, expressivists already accept that (38) entails both factual sentences such as (39) and nonfactual sentences such as (40):

- (38) If it is raining, they are at a local pub.
- (39) Either it is not raining or they are at a local pub.
- (40) If it is raining, they are at a pub.

Hence an expressivist account of probabilistic knowledge ascriptions will resemble extant accounts of other language of subjective uncertainty.

Furthermore, our very motivations for rejecting truth-conditional semantic theories extend to probabilistic knowledge ascriptions. The example of Lucy and the lottery shows that probabilistic knowledge ascriptions prompt just the sort of retraction behavior that motivates

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MacFarlane (2011) to reject a standard semantics for epistemic modals. In addition, eavesdroppers who had read the newspaper would deny your earlier assertion of ‘Lucy knows that she probably lost the lottery’ just as they would deny ‘Lucy probably lost the lottery’ and similar bare epistemic modal sentences. If the latter denials are evidence against truth-conditional theories of bare epistemic modal sentences, as Egan (2007) suggests, then the former denials should be equal evidence against truth-conditional theories of probabilistic knowledge ascriptions. To take one last example, recall that Bennett (2003) and Yalcin (2009) complain that standard truth-conditional theories flout our intuitions about the subject matter of bare epistemic modal sentences. The same goes for probabilistic knowledge ascriptions. For example, (20) intuitively describes what someone realizes about her feelings, not what she realizes about some contextually determined evidence about her feelings:

(20) I realized that I probably liked that guy as more than a friend.

Hence an expressivist account of probabilistic knowledge ascriptions is not only analogous to extant expressivist accounts of factive contexts; it is independently motivated.

It might be objected that an expressivist account of knowledge ascriptions threatens what we have always held valuable about knowledge, or at least threatens the notion that knowledge should be the primary object of interest for epistemology. But as explained in section 4, expressivists may maintain that knowledge is factive, safe, sensitive, and not preserved in Gettier cases. Expressivists may hold that knowledge cannot be analyzed, that it is the norm of assertion, that it is transmitted by testimony, and that it constitutes your evidence. They may even claim that mentioning probabilistic knowledge is essential for the strength of certain explanations. To modify an example from Williamson 2000: a burglar may be more likely to spend all night in a house that probably contains diamonds when he knows it probably contains diamonds, rather than when he merely justifiedly believes it probably contains diamonds. The account of probabilistic knowledge ascriptions defended here is compatible with a wide range of claims that have traditionally been used to characterize the value and primacy of knowledge. In light of these arguments, the burden lies with those who would argue that these claims miss something essential about the mental state of knowledge.

*Epistemology Formalized*5.5. *Probabilistic Quasi-knowledge*

Analyses of probabilistic knowledge in terms of more familiar epistemic notions do not seem forthcoming. But even if they were, one might still endorse the central claims of this paper: that probabilistic knowledge can help us solve several problems and that it can do so without overturning our core intuitions about the nature of knowledge. Some may resist the latter claim on conceptual grounds, however. They may claim that it is an analytic truth that properties of credence distributions simply cannot constitute knowledge. Their resistance raises important metasemantic questions, such as whether it is analytic that knowledge is a relation to propositions, and how we should settle disputes about such analyticity claims. These questions are comparable to questions about plan-laden judgments raised in Gibbard (2003, 235):

Can we, then, sometimes know what to do? When we do, is this real knowledge; is it knowledge in the same sense as with natural features of our surroundings? Knowledge or quasi-knowledge—which it is I won't try saying. In crucial respects, though, plan-laden judgments can at least parallel the clearest and most literal cases of knowledge.

Gibbard adopts a reasonably modest stance about knowledge ascriptions embedding ethical vocabulary. Similarly, it is beyond the scope of this paper to settle metasemantic questions about knowledge ascriptions embedding language of subjective uncertainty. In order to remain as neutral as possible, I shall instead offer a few more modest alternative theses about properties of credence distributions, claims that should be more palatable to conceptually cautious audiences. Alternative thesis one: knowledge is a member of a natural kind of epistemically good mental states, and that natural kind also includes mental states that are relations to properties of credence distributions. Alternative thesis two: beliefs that constitute knowledge have a certain primitive epistemic virtue, and properties of credence distributions can have the very same virtue.

Advocates of these alternative theses may respond to the problem in section 1 by saying that we use 'knows' to relate subjects to epistemically good properties of credence distributions. They may agree that 'knows' ascriptions are factive in the sense of (FACTIVE₂), and this may help them explain why we use the term 'knows' in such ascriptions, despite the fact that the ascriptions do not concern knowledge but rather some other related mental state. Advocates of the alternative theses may also endorse the claim that the quasi-knowledge relation is safe in the sense of (SAFE₂),

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and the fact that quasi-knowledge is factive and safe may help them capture our intuition that there are strong and important similarities between propositional and probabilistic Gettier cases. To sum up: accepting probabilistic knowledge is the simplest and most natural way to resolve the problems in sections 1–2. And the discussion in section 4 demonstrates that one can accept probabilistic knowledge at little cost. This constitutes a positive case for probabilistic knowledge. It is compatible with my having made this case that for some theorists, the cost of accepting probabilistic knowledge may still be high enough to outweigh concerns of simplicity and naturalness. For such theorists, the moral of the present paper is that some properties of credence distributions may be epistemically good in the same way that beliefs are good when they constitute knowledge.

6. Further Applications for Probabilistic Knowledge

In assessing theoretical reasoning, it is natural to talk about whether an agent knows a proposition—for instance, whether an agent knows the premises of some argument, or knows that some rules of inference are valid. A number of recent papers have argued that whether an agent knows a proposition also affects our assessment of the agent's *practical* reasoning. In particular, many have argued that there is an intimate connection between what you know and what you may treat as your reasons for doing something.²⁶ In this section, I briefly discuss principles connecting knowledge and action defended in Hawthorne and Stanley 2008 and Weatherson 2012. Both principles face significant problems. Accepting probabilistic knowledge provides tidy solutions to both problems.

Suppose that Allan has a hunch that he will not get sick this year. He declines reasonably priced health insurance, saying:

- (41) I should decline the health insurance since I am not going to get sick this year.

Intuitively, there is something wrong with Allan declining insurance for the reason he gives in (41), namely that he is not going to get sick this year. Hawthorne and Stanley (2008) argue that Allan should not decline insurance for that reason because he does not know that he is not going to get sick. Abstracting from particular cases, they argue for a general principle

26. For starters, see Unger 1974, Hyman 1999, Fantl and McGrath 2002, Hawthorne 2004, Stanley 2005, and Fantl and McGrath 2007.

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connecting knowledge and rational action: an agent should act only on the basis of reasons that that agent knows.

In response to Hawthorne and Stanley (2008), some have objected that when an action is informed by rational credences, an agent can act for reasons that do not constitute knowledge. For instance, Schiffer (2007, 189) objects that “the following sort of example is very common: you are completely justified in carrying an umbrella even though you don’t know that it will rain but merely believe to degree .4 that it will rain.” To take another example, suppose that Alice has one of ten thousand tickets in a lottery with a \$5 first prize. Alice decides to sell her ticket for a penny, saying:

(42) I should sell my ticket since it is merely slightly likely that my ticket is the winner.

The objection to Hawthorne and Stanley (2008) is that Alice sells her ticket for a perfectly good reason in this case, though the credence that informs her action does not constitute knowledge. Hawthorne and Stanley (2008) respond to this sort of objection by arguing that Alice uses (42) to say that it is merely slightly likely on her evidence that her ticket is the winner, and that this known fact about her evidence constitutes her reason for selling her ticket. But the anticontextualist arguments in section 1 should caution the reader against hastily adopting this response. For example, saying that Alice sells her ticket on the basis of a fact about her own epistemic state seems to misidentify the subject matter of her stated reason for selling her ticket.²⁷

Weatherson 2012 advocates a second principle connecting knowledge and rational action. The central claim is that one may accurately represent a decision problem without representing states of the world to which the agent gives some credence, if the agent knows that those states of the world do not obtain. Hence agents may rationally apply the Sure-Thing Principle to choose an action that dominates alternatives in every possibility consistent with their knowledge. Conversely, if some state of the world is consistent with their knowledge, that state must be included in any appropriate representation of their decisions. According to Weatherson, that is why the Sure-Thing Principle cannot tell Allan to decline insurance: it is consistent with his knowledge that he is going to get sick,

27. For further development of the objection to Hawthorne and Stanley 2008, see Cresto 2010. For instances of the contextualist response, see Hawthorne 2004, 135; Stanley 2005, 10; and Hawthorne and Stanley 2008, 581ff.

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and so declining insurance does not dominate accepting it. In slogan form, the general principle is that “knowledge structures decision problems.”

Trouble is just around the corner for this knowledge-first approach to decision theory. Suppose that Barry is sitting in his apartment when he hears a familiar sound outside. It is his favorite street musician, Beth. He is hurrying down to meet her when a genie appears and offers him a free bet. If he takes the bet and the musician is indeed Beth, he will get \$10. But if it isn't Beth, he will be tortured for several hours. A dilemma ensues. It seems that Barry should decline the bet. But according to the claim that knowledge structures decision problems, if Barry knows that the street musician is Beth, his decision is appropriately represented by a table according to which taking the bet is the dominant option. Weatherson (2012, 83–84) summarizes the problem:

If you accept that the bet should be declined, then it seems to me that there are three options available.

1. Barry never knew that the musician was Beth.
2. Barry did know that the musician was Beth, but this knowledge was destroyed by the genie's offer of the bet.
3. States of the world that are known not to obtain should still be represented in decision problems, so taking the bet is not a dominating option.

Weatherson concludes that in order to avoid skepticism, knowledge-first decision theorists must embrace option (2). That is, they must admit that Barry's knowledge is *interest relative*, depending not only on facts traditionally treated as epistemic but also on practical facts about his situation. Weatherson argues elsewhere that epistemic interest relativity is not as bad as it first seems (see Weatherson 2011), so that this is not a costly outcome for knowledge-first decision theorists. But many have argued that interest relativity is extremely counterintuitive, and they may count the above argument as a strong reason to reject a knowledge-first decision theory.²⁸

Both Schiffer and Weatherson raise significant problems for knowledge-based norms of action and decision making. But in both

28. Even those who accept interest relativity often admit that it is an extremely counterintuitive claim. For further discussion, see Stanley 2007, 168; Fantl and McGrath 2009, 186–87; and DeRose 2009, 189.

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cases, the advocate of probabilistic knowledge has an easy way out. Suppose that Hawthorne and Stanley accept that properties of credence distributions may constitute knowledge. Then they may endorse a probabilistic-knowledge-based norm of action: agents must act on the basis of properties of their credence distribution that constitute knowledge. The case of Alice is consistent with this norm: Alice may sell her ticket, as long as she *knows* that it is merely slightly likely that it is the winner. That is, Alice may sell her ticket if her very low credence that it is the winner constitutes knowledge.

One may similarly replace the knowledge-based norms in Weatherston 2012 with less problematic norms concerning probabilistic knowledge. For instance, knowledge-first decision theorists may claim that if an action has maximal expected value according to properties of your credence distribution that constitute knowledge, you should perform that action. For example: in representing Barry's initial decision to go downstairs, it is permissible to appeal to the premise that it is at least .8 likely that Beth is outside, precisely because this premise is known by Barry. Since this premise entails that going downstairs has maximal expected value, Barry should go downstairs. On the other hand, in representing the decision to take the genie's bet, it is not permissible to appeal to the premise that it is at least .9999 likely that Beth is outside, precisely because this premise is not known by Barry. In both cases, what Barry should do depends on what probabilistic knowledge he has. But that does not mean that his probabilistic knowledge is interest relative. Barry does not lose his knowledge that it is at least .8 likely that Beth is outside when the genie offers him a bet. His declining the bet is rational because he lacks much stronger probabilistic knowledge, knowledge that he never had to begin with.

The aim of the present discussion is modest. I have defended knowledge-based norms of action and decision making against the challenges articulated above. But I have not argued that we must accept any knowledge-based norms. For instance, I have not responded to a third challenge for knowledge-based norms recently raised by Brown (2008), Littlejohn (2009), and Neta (2009). These authors argue that mere justified true belief suffices for rational agency. In a similar spirit, a subjective Bayesian might argue that maximizing expected value suffices for rational agency, regardless of whether your credences are intuitively justified. This third challenge is quite distinct from the charge that knowledge-based norms neglect action based on credences, or that they force us to accept interest relativity. It would be naïve to think that

accepting probabilistic knowledge could answer every challenge faced by knowledge-based accounts of rational agency; some challenges should be answered by other means. However, insofar as the third challenge succeeds in undermining knowledge-based accounts of rational agency, some motivation for accepting probabilistic knowledge is also undermined. In the spirit of preserving that motivation, it is worth reviewing how advocates of knowledge-based norms might respond to the third challenge.

Here is one respectable response: action based on a justified true full belief can seem intuitively worse than action based on knowledge. There is an intuitive sense in which it is not perfectly okay for Henry to act on his belief that he sees a barn as he drives his son through fake barn country. There is a sense in which something goes wrong when Henry tells his son that he sees a barn, for instance. And there is a sense of ‘should’ that you may use to criticize his action. The norms expressed with that sense of ‘should’ sanction different actions than traditional Bayesian norms of rational action. But both sorts of norms spell out legitimate and valuable notions of rationality. The same holds for norms governing action based on credences. There is a sense in which it is not perfectly okay for Henry to act on his high credence that he sees a barn. There is a sense in which something goes wrong if Henry decides to bet a lot of money on the claim that he is looking at a barn, for instance. And knowledge-based norms of action spell out a notion of rationality appropriate to that normative intuition.

To sum up: the theory that properties of credence distributions can constitute knowledge allows us to solve significant problems. I argued in sections 1–2 that this theory allows us to reconcile work in traditional epistemology with necessarily sophisticated models of assertion and updating. And here I have sketched how the theory allows us to develop knowledge-based norms of action and decision making without accepting either contextualist accounts of reasons statements or the counter-intuitive claim that knowledge is interest relative. Insofar as knowledge-based norms of action and decision making are compelling, such arguments constitute reasons to accept probabilistic knowledge.

This is just the beginning. Once we accept probabilistic knowledge, we can construct probabilistic analogues of a number of traditional claims about the role of knowledge, such as that knowledge is the aim of belief, that it is the norm of assertion, and that it constitutes your evidence. Advocates of probabilistic knowledge may also construct theories about epistemically virtuous credences that are informed by

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traditional theories of propositional knowledge. For instance: foundationalists may say that properties of your credence distribution constitute knowledge just in case they are justified by certain constraints directly imposed on your credences; reliabilists may say that properties constitute knowledge just in case they result from cognitive processes that generally produce accurate credences; and so on. In short, accepting probabilistic knowledge allows us to fully explore nascent attempts to merge traditional epistemological theories and formal representations of doxastic states. And these benefits are achieved at little cost to our intuitions about core features of knowledge. The Bayesian can accommodate both the assumptions and the insights of the traditional study of epistemology. Knowledge may be first, but that does not mean that credences must be second.

References

- Adams, Ernest W. 1965. "A Logic of Conditionals." *Inquiry* 8(1–4): 166–97.
 ———. 1975. *The Logic of Conditionals*. Dordrecht: D. Reidel.
- Alston, W. 1988. "An Internalist Externalism." *Synthese* 74(3): 265–83.
- Beaver, David I. 2001. *Presupposition and Assertion in Dynamic Semantics*. Stanford: CSLI Publications.
- Belzoni, Giovanni. 1820. *Narrative of the Operations and Recent Discoveries within the Pyramids, Temples, Tombs, and Excavations in Egypt and Nubia; and of a Journey to the Coast of the Red Sea, in Search of the Ancient Berenice; and Another to the Oasis of Jupiter Ammon*. London: John Murray.
- Bennett, Jonathan. 2003. *A Philosophical Guide to Conditionals*. Oxford: Oxford University Press.
- Blackburn, Simon. 1996. "Securing the Nots: Moral Epistemology for the Quasi-Realist." In *Moral Knowledge?: New Readings in Moral Epistemology*, ed. Walter Sinnott-Armstrong and Mark Timmons, 82–100. New York: Oxford University Press.
- . 1998. *Ruling Passions: A Theory of Practical Reasoning*. Oxford: Oxford University Press.
- Brown, Jessica. 2008. "Subject-Sensitive Invariantism and the Knowledge Norm for Practical Reasoning." *Nous* 42(2): 167–89.
- Chrisman, Matthew. 2010. "Constructivism, Expressivism and Ethical Knowledge." *International Journal of Philosophical Studies* 18(3): 331–53.
- Collins, Robin. 1999. "A Scientific Argument for the Existence of God: The Fine-Tuning Design Argument." In *Reason for the Hope Within*, ed. Michael Murray, 47–75. Grand Rapids, MI: Eerdmans.

- Comocrush. 2009. Comment in discussion forum: Celebrity Rehab Sober House, Televisionwithoutpity.com. Posted February 3. forums.televisionwithoutpity.com/index.php?showtopic=3180931&st=885.
- Conee, Earl, and Richard Feldman. 2010. "Earl Conee and Richard Feldman." In *A Companion to Epistemology*, ed. Jonathan Dancy, Ernest Sosa, and Matthias Steup, 123–30. Oxford: Blackwell.
- Cresto, Eleonora. 2010. "On Reasons and Epistemic Rationality." *Journal of Philosophy* 107(6): 326–30.
- de Finetti, Bruno. [1937] 1993. "Foresight: Its Logical Laws, Its Subjective Sources," trans. Henry E. Kyburg, Jr. In *Breakthroughs in Statistics: Volume 1: Foundations and Basic Theory*, ed. Samuel Kotz and Norman L. Johnson, New York: Wiley.
- DeRose, Keith. 1991. "Epistemic Possibilities." *Philosophical Review* 100(4): 581–605.
- . 2009. *The Case for Contextualism*. Oxford: Oxford University Press.
- Diaconis, Persi, and Sandy L. Zabell. 1982. "Updating Subjective Probability." *Journal of the American Statistical Association* 77(380): 822–30.
- Draper, Paul. 1989. "Pain and Pleasure: An Evidential Problem for Theists." *Nous* 23(3): 331–50.
- Dutton, D. G., and A. P. Aron. 1974. "Some Evidence for Heightened Sexual Attraction under Conditions of High Anxiety." *Journal of Personality and Social Psychology* 30(4): 510–7.
- Edgington, Dorothy. 1995. "On Conditionals." *Mind* 104(414): 235–329.
- Egan, Andy. 2007. "Epistemic Modals, Relativism, and Assertion." *Philosophical Studies* 133(1): 1–22.
- Egan, Andy, John Hawthorne, and Brian Weatherson. 2005. "Epistemic Modals in Context." In *Contextualism in Philosophy*, ed. Gerhard Preyer and Georg Peter, 131–69. Oxford: Oxford University Press.
- Fantl, Jeremy, and Matthew McGrath. 2002. "Evidence, Pragmatics, and Justification." *Philosophical Review* 111(1): 67–94.
- . 2009. "Critical Study of John Hawthorne, *Knowledge and Lotteries* and Jason Stanley, *Knowledge and Practical Interests*." *Nous* 43(1): 178–92.
- . 2007. "On Pragmatic Encroachment in Epistemology." *Philosophy and Phenomenological Research* 75(3): 558–89.
- von Fintel, Kai, and Anthony S. Gillies. 2011. "'Might' Made Right." In *Epistemic Modality*, ed. Andy Egan and Brian Weatherson, 108–30. Oxford: Oxford University Press.
- Foley, Richard. 1993. *Working without a Net*. Oxford: Oxford University Press.
- French, S. 1985. "Group Consensus Probability Distributions: A Critical Survey." In *Bayesian Statistics 2*, ed. J. M. Bernardo, et al., 183–97. Amsterdam: North-Holland.
- Fumerton, Richard. 2006. *Epistemology*. Oxford: Blackwell.

Epistemology Formalized

- Gettier, Edmund. 1963. "Is Justified True Belief Knowledge?" *Analysis* 23(6): 121–3.
- Geurts, Bart. 2005. "Entertaining Alternatives: Disjunctions as Modals." *Natural Language Semantics* 13(4): 383–410.
- Gibbard, Allan. 1981. "Two Recent Theories of Conditionals." In *Ifs: Conditionals, Belief, Decision, Chance, and Time*, ed. William L. Harper, Robert Stalnaker, and Glenn Pearce, 211–47. Dordrecht: D. Reidel.
- . 2003. *Thinking How to Live*. Cambridge, MA: Harvard University Press.
- Gillies, Anthony S. 2004. "Epistemic Conditionals and Conditional Epistemics." *Nous* 38(4): 585–616.
- . 2009. "On Truth-Conditions for *If* (but Not Quite Only *If*)." *Philosophical Review* 118(3): 325–49.
- Goldman, Alvin. 1976. "Discrimination and Perceptual Knowledge." *Journal of Philosophy* 73(20): 771–91.
- . 1986. *Epistemology and Cognition*. Cambridge, MA: Harvard University Press.
- Grünwald, Peter, and Joseph Y. Halpern. 2003. "Updating Probabilities." *Journal of Artificial Intelligence Research* 19: 243–78.
- Hacking, Ian. 1967. "Possibility." *Philosophical Review* 76(2): 143–68.
- Hájek, Alan. 2003. "What Conditional Probability Could Not Be." *Synthese* 137(3): 273–323.
- Hall, Ned. 1994. "Correcting the Guide to Objective Chance." *Mind* 103(412): 505–18.
- Hall, Ned, and Alan Hájek. 1994. "The Hypothesis of the Conditional Construal of Conditional Probability." In *Probabilities and Conditionals: Belief Revision and Rational Decision*, ed. Bryan Skyrms and Ellery Eells, 75–110. Cambridge: Cambridge University Press.
- Hawthorne, John. 2004. *Knowledge and Lotteries*. Oxford: Oxford University Press.
- Hawthorne, John, and Jason Stanley. 2008. "Knowledge and Action." *Journal of Philosophy* 105: 571–90.
- Hazlett, Allan. 2010. "The Myth of Factive Verbs." *Philosophy and Phenomenological Research* 80(3): 497–522.
- Hintikka, Jaakko. 1962. *Knowledge and Belief: An Introduction to the Logic of Two Notions*. Ithaca, NY: Cornell University Press.
- Hostetter, Jeff. 2012. Exterior Painting—How Long Should It Last? JD Hostetter & Associates website, www.jdhostetter.com/news/detail.asp?news_id=3 (accessed August 20).
- Huber, Franz, and Christoph Schmidt-Petri, eds. 2009. *Degrees of Belief*. Dordrecht: Springer.
- Huddleston, Rodney, and Geoffrey K. Pullum, eds. 2002. *The Cambridge Grammar of the English Language*. Cambridge: Cambridge University Press.

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- Hughes, George, and Max Cresswell. 1996. *A New Introduction to Modal Logic*. New York: Routledge.
- Hyman, John. 1999. "How Knowledge Works." *Philosophical Quarterly* 49(197): 433–51.
- Jaynes, E. T. 1978. "Where Do We Stand on Maximum Entropy." In *The Maximum Entropy Formalism*, ed. R. Levine and M. Tribus, 15–118. Cambridge, MA: MIT Press.
- Jeffrey, Richard C. 1965. "Probability Kinematics." In *The Logic of Decision*, 164–83. Chicago: University of Chicago Press.
- . 1983. *The Logic of Decision*. 2nd ed. Chicago: University of Chicago Press.
- Joyce, James M. 1999. *The Foundations of Causal Decision Theory*. Oxford: Oxford University Press.
- . 2005. "How Probabilities Reflect Evidence." *Philosophical Perspectives* 19(1): 153–78.
- Kamp, Hans. 1974. "Free Choice Permission." *Proceedings of the Aristotelian Society* 74: 57–74.
- Kaplan, David. 1989. "Afterthoughts." In *Themes from Kaplan*, ed. Joseph Almog, John Perry, and Howard Wettstein, 565–614. Oxford: Oxford University Press.
- Karttunen, Lauri. 1974. "Presupposition and Linguistic Context." *Theoretical Linguistics* 1(1–3): 181–94.
- Kiparsky, Paul, and Carol Kiparsky. 1970. "Fact." In *Semantics*, ed. D. Steinberg and L. Jakobovits, 345–69. Cambridge: Cambridge University Press.
- Kratzer, Angelika. 1977. "What 'Must' and 'Can' Must and Can Mean." *Linguistics and Philosophy* 1(3): 337–55.
- . 1978. *Semantik der Rede: Kontexttheorie, Modalwörter, Konditionalsätze*. Königstein: Scriptor.
- . 1981a. "The Notional Category of Modality." In *Words, Worlds, and Contexts: New Approaches in Word Semantics*, ed. Hans Jürgen Eikmeyer and Hannes Rieser, 38–74. Berlin: W. de Gruyter.
- . 1981b. "Partition and Revision: The Semantics of Counterfactuals." *Journal of Philosophical Logic* 10: 201–16.
- . 1986. "Conditionals." In *Semantics: An International Handbook of Contemporary Research*, ed. Arnim von Stechow and Dieter Wunderlich, 651–56. Berlin: W. de Gruyter.
- . 2010. "Hunting Down the Material Conditional." Paper presented at the Rutgers Semantics Workshop, Rutgers University.
- Kvanvig, Jonathan. 2010. "Epistemic Justification." In *Routledge Companion to Epistemology*, ed. Sven Bernecker and Duncan Pritchard, 25–36. New York: Routledge.
- Kyburg, H., and C. Teng. 2001. *Uncertain Inference*. Cambridge: Cambridge University Press.

Epistemology Formalized

- Lewis, David K. 1976. "Probabilities of Conditionals and Conditional Probabilities." In *Philosophical Papers 2*, 133–56. Oxford: Oxford University Press.
- Littlejohn, Clayton. 2009. "Must We Act Only on What We Know?" *Journal of Philosophy* 106(8): 463–74.
- MacFarlane, John. 2011. "Epistemic Modals Are Assessment-Sensitive." In *Epistemic Modality*, ed. Andy Egan and Brian Weatherson, 144–78. Oxford: Oxford University Press.
- Maher, Patrick. 2004. "Probability Captures the Logic of Scientific Confirmation." In *Contemporary Debates in Philosophy of Science*, ed. Chris Hitchcock, 69–93. Oxford: Blackwell.
- McGee, Vann. 1994. "Learning the Impossible." In *Probabilities and Conditionals: Belief Revision and Rational Decision*, ed. Bryan Skyrms and Ellery Eells, 179–200. Cambridge: Cambridge University Press.
- Moore, G. E. 1962. *Commonplace Book: 1919–1953*. London: George Allen and Unwin.
- Moss, Sarah. n.d. "On the Pragmatics of Epistemic Modals." Unpublished manuscript.
- Neta, Ram. 2009. "Treating Something as a Reason for Action." *Nous* 43(4): 684–99.
- Nozick, Robert. 1981. *Philosophical Explanations*. Cambridge, MA: Harvard University Press.
- peedyCakey. 2009. I know I can't have him, so what do I do? DearCupid.ORG advice question. Posted October 21. www.dearcupid.org/question/i-know-i-cant-have-him-so-what.html.
- Plantinga, Alvin. 1996. "Respondeo." In *Warrant in Contemporary Epistemology: Essays in Honor of Plantinga's Theory of Knowledge*, ed. Jonathan Kvanvig, 307–78. Lanham, MD: Rowman and Littlefield.
- Pollock, John, and Joseph Cruz. 1999. *Contemporary Theories of Knowledge*. Lanham, MD: Rowman and Littlefield.
- Pritchard, Duncan. 2005. *Epistemic Luck*. Oxford: Clarendon.
- Quine, W. V. O. 1951. "Two Dogmas of Empiricism." In *From a Logical Point of View*, 2nd ed, 20–46. Cambridge, MA: Harvard University Press.
- Ramsey, F. P. 1926. "Truth and Probability." In *Philosophical Papers*, ed. D. H. Mellor, 52–94. Cambridge: Cambridge University Press.
- Ridge, Michael. 2007. "Epistemology for Ecumenical Expressivists." *Proceedings of the Aristotelian Society* 81(1): 83–108.
- Schiffer, Stephen R. 2007. "Interest-Relative Invariantism." *Philosophy and Phenomenological Research* 75(1): 188–95.
- Schnieder, Benjamin. 2010. "Expressivism concerning Epistemic Modals." *Philosophical Quarterly* 60(240): 601–15.
- Schroeder, Mark. 2008. *Being For: Evaluating the Semantic Program of Expressivism*. Oxford: Oxford University Press.

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- . n.d. “Hard Cases for Combining Expressivism and Deflationist Truth: Conditionals and Epistemic Modals.” Unpublished manuscript.
- Simons, Mandy. 2005. “Dividing Things Up: The Semantics of *Or* and the Modal/*Or* Interaction.” *Natural Language Semantics* 13(3): 271–316.
- Skyrms, Bryan. 1987. “Updating, Supposing, and MAXENT.” *Theory and Decision* 22(3): 225–46.
- Stalnaker, Robert C. 1970. “Probability and Conditionals.” *Philosophy of Science* 37(1): 64–80.
- Stanley, Jason. 2005. *Knowledge and Practical Interests*. Oxford: Oxford University Press.
- . 2007. “Précis of Knowledge and Practical Interests.” *Philosophy and Phenomenological Research* 75(1): 168–72.
- Stephenson, Tamina. 2007. “Judge Dependence, Epistemic Modals, and Predicates of Personal Taste.” *Linguistics and Philosophy* 30(4): 487–525.
- Stjernberg, Fredrik. 2009. “Restricting Factiveness.” *Philosophical Studies* 146(1): 29–48.
- Swanson, Eric. 2006. “Interactions with Context.” PhD diss., MIT.
- . 2010. “On Scope Relations between Quantifiers and Epistemic Modals.” *Journal of Semantics* 27(4): 529–40.
- Teller, Paul. 1972. “Epistemic Possibility.” *Philosophia* 2(4): 303–20.
- Unger, Peter. 1974. “Two Types of Skepticism.” *Philosophical Studies* 25(2): 77–96.
- van Fraassen, Bas C. 1980. “Rational Belief and Probability Kinematics.” *Philosophy of Science* 47(2): 165–87.
- . 1981. “A Problem for Relative Information Minimizers in Probability Kinematics.” *British Journal for the Philosophy of Science* 32(4): 375–79.
- . 1989. *Laws and Symmetry*. Oxford: Oxford University Press.
- van Inwagen, Peter. 1996. “Reflections on the Chapters by Draper, Russell, and Gale.” In *The Evidential Argument from Evil*, ed. Daniel Howard-Snyder, 219–43. Bloomington: Indiana University Press.
- Veltman, Frank. 1985. “Logics for Conditionals.” PhD diss., University of Amsterdam.
- . 1996. “Defaults in Update Semantics.” *Journal of Philosophical Logic* 25(3): 221–61.
- Weatherston, Brian. 2005. “Can We Do Without Pragmatic Encroachment?” *Philosophical Perspectives* 19(1): 417–43.
- . 2008. “Attitudes and Relativism.” *Philosophical Perspectives* 22(1): 527–44.
- . 2011. “Defending Interest-Relative Invariantism.” *Logos and Episteme* 2(4): 591–609.
- . 2012. “Knowledge, Bets, and Interests.” In *Knowledge Ascriptions*, ed. Jessica Brown and Mikkel Gerken, 75–103. Oxford: Oxford University Press.

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- Webb, James, et al. 2007. *A Parent's Guide to Gifted Children*. Scottsdale: Great Potential Press.
- Willem, Jan. 2005. Comment in discussion forum: fly for Jan Willem Department? Diptera.info website. Posted October 5. www.diptera.info/forum/viewthread.php?forum_id=5&thread_id=782.
- Williamson, Timothy. 2000. *Knowledge and Its Limits*. Oxford: Oxford University Press.
- . 2004. "Summary." *Philosophical Books* 45(4): 283–5.
- Yalcin, Seth. 2007. "Epistemic Modals." *Mind* 116(464): 983–1026.
- . 2009. "Credal Expressivism." Paper presented at the Arché Expressivism Workshop, University of St Andrews.
- . 2011. "Nonfactualism about Epistemic Modality." In *Epistemic Modality*, ed. Andy Egan and Brian Weatherson, 295–332. Oxford: Oxford University Press.
- Zimmermann, Thomas Ede. 2000. "Free Choice Disjunction and Epistemic Possibility." *Semantics* 8(4): 255–90.