50. SHORT RESPONSE  Maddie works at Game Exchange. They are having a sale as shown.

<table>
<thead>
<tr>
<th>Item</th>
<th>Price (dollars)</th>
<th>Special</th>
</tr>
</thead>
<tbody>
<tr>
<td>video games</td>
<td>$20</td>
<td>Buy 2 get 1 Free</td>
</tr>
<tr>
<td>DVDs</td>
<td>$15</td>
<td>Buy 1 get 1 Free</td>
</tr>
</tbody>
</table>

Her employee discount is 15%. If sales tax is 7.25%, how much does she spend for a total of 4 video games?

51. Solve \( \frac{1}{2}x + 7 = \frac{3}{15}x - 3 \).

A. \(-16\)  
B. \(-14\)  
C. \(-10\)  
D. \(-6\)

52. 5m + 6 = -4
53. \(-1 = 7 + 3c\)
54. \(\frac{1}{2}z + 7 = 16 - \frac{3}{5}z\)
55. \(\frac{5}{2}x + 6 = \frac{3}{2}x + 10\)
56. \(\frac{9}{2} = 3 - x\)
57. \(9 + \frac{x}{5} = 6\)

58. WORLD RECORDS  In 1998, Winchell's House of Donuts in Pasadena, California, made the world's largest donut. It weighed 5000 pounds and had a circumference of 296.3 feet. What was the donut's diameter to the nearest tenth? (Hint: C = \(\pi\diameter\)) (Lesson 2-2)

59. 200  At a zoo, the cost of admission is posted on the sign. Find the cost of admission for two adults and two children. (Lesson 1-4)

Find the value of \(x\). Then name the property used in each step. (Lesson 1-5)

60. 25n = 25  
61. \(n \cdot 1 = \frac{2}{5} \cdot \frac{5}{2}\)
62. \(12 \cdot p = 12 \cdot 6\)
63. \(n + 0 = \frac{3}{2}\)  
64. \(4 \cdot \frac{1}{3} = n\)
65. \((10 - 8)(5) = 20\)
66. \((10 - 8)(5) = 20\)

Skills Review

Translate each sentence into an equation.
66. Twice a number \(t\) decreased by eight equals seventy. \(2t - 8 = 70\)
67. Five times the sum of \(m\) and \(k\) is the same as seven times \(k\). \(5(m + k) = 7k\)
68. Half of \(p\) is the same as \(p/2\) minus 3. \(\frac{p}{2} - p = -3\)

Evaluate each expression.
69. \((-9) + (-14) = -5\)
70. \((-10 + 20) = 10\)
71. \(-15 + 9 \leq -2(\frac{1}{2})\)
72. \(5(14) = 70\)
73. \(-55 + (-5) = -60\)
74. \(-25(-5) = 125\)

1. Absolute Value Expressions  Expressions with absolute value define an upper and lower range in which a value must lie. Expressions involving absolute value can be evaluated using the given value for the variable.

Example 1  Expressions with Absolute Value

Evaluate \(|m + 6| - 14\) if \(m = 4\).

\(|m + 6| - 14 = |4 + 6| - 14 = 10 - 14 = -4\)

2. Absolute Value Equations  The margin of error in the example at the top of the page is an example of absolute value. The distance between -66 and 69 on a number line is the same as the distance between -63 and 66.

There are three types of open sentences involving absolute value, \(|x| = x, |x| < x,\) and \(|x| > x\). In this lesson, we will consider only the first type. Look at the equation \(|x| = 4\). This means that the distance between 0 and \(x\) is 4.

If \(|x| = 4\), then \(x = -4\) or \(x = 4\). Thus, the solution set is \((-4, 4)\).

For each absolute value equation, we must consider both cases. To solve an absolute value equation, first isolate the absolute value on one side of the equals sign if it is not already by itself.
**Key Concept: Absolute Value Equations**

Words: When solving equations that involve absolute values, there are two cases to consider.

- **Case 1:** The expression inside the absolute value symbol is positive or zero.
- **Case 2:** The expression inside the absolute value symbol is negative.

Symbols: For any real numbers \(a\) and \(b\), if \(|a| = b\) and \(b \geq 0\), then \(a = b\) or \(-a = b\).

Example: \(|d| = 10\), so \(d = 10\) or \(d = -10\).

**Example 2: Solve Absolute Value Equations**

Solve each equation. Then graph the solution set.

a. \(|f + 5| = 17\)
   
   **Case 1**
   
   \[f + 5 = 17\]
   \[f = 12\]

   **Case 2**
   
   \[f + 5 = -17\]
   \[f = -22\]

   
   
   

b. \(|b - 1| = -3\)

   \[b - 1 = -3\] means the distance between \(b\) and 1 is \(-3\). Since distance cannot be negative, the solution is the empty set \(\emptyset\).

**Guided Practice**

2A. \(|y + 2| = 4\)  
2B. \(|3n - 4| = -1\)

Absolute value equations occur in real-world situations that describe a range within which a value must lie.

**Real-World Example 1: Solve an Absolute Value Equation**

**SNAKES** The temperature of an enclosure for a pet snake should be about 80°F, give or take 5°F. Find the maximum and minimum temperatures.

You can use a number line to solve.

- The distance from 70 to 75 is 5 units.
- The distance from 65 to 85 is 10 units.

The solution set is [75, 85]. The maximum and minimum temperatures are 85°F and 75°F.

**Guided Practice**

3. **ICE CREAM** Ice cream should be stored at 5°F with an allowance for 3°F. Write and solve an equation to find the maximum and minimum temperatures at which the ice cream should be stored.

When given two points on a graph, you can write an absolute value equation for the graph.

**Example 4: Write an Absolute Value Equation**

Write an equation involving absolute value for the graph.

- Find the point midway between two points, add the values together and divide by 2. For example, 4.11 = 19 = 30, 30 + 2 = 15. So 15 is the point halfway between 11 and 19.

- The midpoint is 15.5.

- The distance from 15 to 11 is 4 units.
- The distance from 19 to 15 is 4 units.

So an equation is \(|x - 15.5| = 4\).

**Guided Practice**

4. Write an equation involving absolute value for the graph.

**Check Your Understanding**

Example 1

Evaluate each expression if \(f = 3\), \(g = -4\), and \(h = 5\).

1. \(|3 - h| + 13\)
2. \(16 - |g + 9|\)
3. \(|f + g| - h\)

Example 2

Solve each equation. Then graph the solution set.

4. \(|a + 7| = 5\)
5. \(|3z - 3| = 9\)
6. \(|4r - 11| = -6\)
7. \(|b + 4| = 2\)
8. \(|22 - 4| = 8\)
9. \(|5h + 2| = -8\)

Example 3

10. **FINANCIAL LITERACY** For a company to invest in a product, they must believe they will receive a 12% return on investment (ROI) plus minus 3%. Write an equation to find the least and the greatest ROI they believe they will receive.

Example 4

Write an equation involving absolute value for each graph.

- The graph shows a point at \((-4, -2)\) and \((0, 0)\).
- The line segment is 8 units long.

- The slope is \(-1\).

- The equation is \(|y - x| = 8\).
Practice and Problem Solving

Example 1 Evaluate each expression if \( a = -2, b = -3, c = 2, x = 2.1, y = 3, \) and \( z = -4.2. \)

13. \( 2x + z + 2y \)
14. \( 4a - \lfloor 3b + 2c \rfloor \)
15. \( -5a + c + |3y + 2z| \)
16. \( -a + |2x - z| \)
17. \( |y - 2x| - 3 \)
18. \( 3|3b - 8c| - 3 \)
19. \( |2x - z| + 6b \)
20. \( -3|x| + 2(a + y) \)
21. \( -4|c - 3| + 2|z - a| \)

Example 2 Solve each equation. Then graph the solution set.

22. \( |n - 3| = 5 \)
23. \( |y + 10| = 1 \)
24. \( |v - 2| = -5 \)
25. \( |4x - 8| = 0 \)
26. \( |8x + 5| = 21 \)
27. \( |6y - 7| = -1 \)
28. \( |\frac{1}{3} x + 5| = -3 \)
29. \( |-2y + 6| = 6 \)
30. \( |\frac{1}{3} |x| - 3| = 9 \)

Example 4 Write an equation involving absolute value for each graph.

33. \( \text{Graph} \)
34. \( \text{Graph} \)
35. \( \text{Graph} \)
36. \( \text{Graph} \)

Solve each equation. Then graph the solution set.

37. \( \frac{1}{2} b - 2 = 10 \)
38. \( |-4d + 6| = 12 \)
39. \( |5f - 3| = 12 \)
40. \( 2|x| - 3 = 8 \)
41. \( 4 - 3|q| = 10 \)
42. \( \frac{1}{2} |p| + 12 = 14 \)

43. Sense-Making The 4 x 400 relay is a race where 4 runners take turns running 400 meters, or one lap around the track.
   a. If a runner runs the first leg in 52 seconds plus or minus 2 seconds, write an equation to find the fastest and slowest times.
   b. If the runners of the second and third legs run their laps in 53 seconds plus or minus 1 second, write an equation to find the fastest and slowest times.
   c. Suppose the runner of the fourth leg is the fastest on the team. If he runs an average of 50.5 seconds plus or minus 1.5 seconds, what are the team's fastest and slowest times?

44. Fashion To allow for a model's height, a designer is willing to use models that require him to change hems either up or down 2 inches. The length of the skirts is 20 inches.
   a. Write an absolute value equation that represents the length of the skirts.
   b. What is the range of the lengths of the skirts?
   c. If a 20-inch skirt was fitted for a model that is 5 feet 9 inches tall, will the designer use a 6-foot-tall model?

45. Precision Speedometer accuracy can be affected by many details such as tire diameter and axle ratio. For example, there is variation of \( \pm 3 \) miles per hour when calibrated at 50 miles per hour.
   a. What is the range of actual speeds of the car if calibrated at 50 miles per hour?
   b. A speedometer calibrated at 45 miles per hour has an accepted variation of \( \pm 1 \) mile per hour. What can we conclude from this?

Write an equation involving absolute value for each graph.

46. \( \text{Graph} \)
47. \( \text{Graph} \)
48. \( \text{Graph} \)
49. \( \text{Graph} \)
50. \( \text{Graph} \)
51. \( \text{Graph} \)

52. Music A CD will record an hour and a half of music plus or minus 3 minutes for time between tracks.
   a. Write an absolute value equation that represents the recording time.
   b. What is the range of time in minutes that the CD could run?
   c. Graph the possible times on a number line.

53. Acoustics The Red Rocks Amphitheater located in the Red Rock Park near Denver, Colorado, is the only naturally occurring amphitheater. The acoustic qualities here are such that a maximum of 20,000 people, plus or minus 1000, can hear natural voices clearly.
   a. Write an equation involving an absolute value that represents the number of people that can hear natural voices at Red Rocks Amphitheater.
   b. Find the maximum and minimum number of people that can hear natural voices clearly in the amphitheater.
54. **BOOK CLUB** The members of a book club agree to read within ten pages of the last page of the chapter. The chapter ends on page 203.
   a. Write an absolute value equation that represents the pages where club members could stop reading.
   b. Write the range of the pages where the club members could stop reading.

55. **SCHOOL** Teams from Washington and McKinley High Schools are competing in an academic challenge. A correct response on a question earns 10 points and an incorrect response loses 10 points. A team earns 0 points on an unattempted question. There are 5 questions in the math section.
   a. What are the maximum and minimum scores a team can earn on the math section?
   b. Suppose the McKinley team has 160 points at the start of the math section. Write and solve an equation that represents the maximum and minimum scores the team could have at the end of the math section.
   c. What are all of the possible scores that a school can earn on the math section?

### H.O.T. Problems Use Higher-Order Thinking Skills

56. **OPEN ENDED** Describe a real-world situation that could be represented by the absolute value equation $|x - 4| = 10$.

57. **STRUCTURE** Determine whether the following statements are sometimes, always, or never true. If $c$ is an integer. Explain your reasoning.
   - The value of $|x + 1|$ is greater than zero.
   - The solution of $|x + c| = 0$ is greater than zero.
   - The inequality $|x| + c < 0$ has no solution.
   - The value of $|x + c| + c$ is greater than zero.

58. **REASONING** Explain why an absolute value can never be negative.

59. **CHALLENGE** Use the sentence $x = 7 + 4.6$.
   a. Describe the values of $x$ that make the sentence true.
   b. Translate the sentence into an equation involving absolute value.

60. **ERROR ANALYSIS** Alex and Wesley are solving $|x + 5| = -3$. Is either of them correct? Explain your reasoning.
   - Alex
     
     $|x + 5| = 3$  
     $x + 5 = 3$  
     $-5 = -5$  
     $x = -2$
   - Wesley
     
     $|x + 5| = -3$  
     The solution is not valid.

61. **WRITING IN MATH** Explain why there are either two, one, or no solutions for absolute value equations. Demonstrate an example of each possibility.

### Standardized Test Practice

65. Which equation represents the second step of the solution process?
   Step 1: $4(2x + 7) - 6 = 3x$
   Step 2: 
   Step 3: $5x + 28 - 6 = 0$
   Step 4: $5x = -22$
   Step 5: $x = -4.4$
   a. $4(2x - 6) + 7 = 3x$
   b. $4(2x + 1) = 3x$
   c. $8x + 7 - 6 = 3x$
   d. $8x + 28 - 6 = 3x$
   
66. **GEOMETRY** The area of a circle is $25\pi$ square centimeters. What is the circumference?

$$ F \ 625\pi \text{cm} \\
G \ 50\pi \text{cm} \\
H \ 25\pi \text{cm} \\
J \ 10\pi \text{cm} $$

67. Tanya makes $5 an hour and 15% commission of the total dollar value on cosmetics she sells. Suppose Tanya’s commission is increased to 17%. How much money will she make if she sells $300 worth of product and works 30 hours?
   a. $201
   b. $226
   c. $235
   d. $283

68. **EXTENDED RESPONSE** John’s mother has agreed to take him driving every day for two weeks. On the first day, John drives for 20 minutes. Each day after that, John drives 5 minutes more than the day before.
   a. Write an expression for the minutes John drives on the nth day.
   b. For how many minutes will John drive on the last day? Show your work.
   c. John’s driver’s education teacher requires that each student drive for 30 hours with an adult outside of class. Will John’s sessions with his mother fulfill this requirement?

### Spiral Review

Write and solve an equation for each sentence. (Lesson 2-4)

69. One half of a number increased by 16 is four less than two thirds of the number.

70. The sum of one half of a number and 6 equals one third of the number.

71. **SHOE** If $t$ represents the length of a man’s foot in inches, the expression $2t - 12$ can be used to estimate his shoe size. What is the approximate length of a man’s foot if he wears a size 8? (Lesson 2-5)

### Skills Review

Write an equation for each problem. Then solve the equation.

72. Seven times a number equals $-84$. What is the number?

73. Two fifths of a number equals $-24$. Find the number.

74. Negative 117 is nine times a number. Find the number.

75. Twelve is one fifth of a number. What is the number?