1-6 Relations

**Represent relations.** The deeper in the ocean you are, the greater pressure is on your body. This is because there is more water over you. The force of gravity pulls the water weight down, creating a greater pressure.

The equation that relates the total pressure of the water to the depth is \( P = \rho gh \), where \( P \) is the pressure, \( \rho \) is the density of water, \( g \) is the acceleration due to gravity, and \( h \) is the height of water above you.

**Interpret graphs of relations.**

**New Vocabulary**
coordinate system
coordinate plane
\( x \)- and \( y \)-axis
origin
ordered pair \( x \)- and \( y \)-coordinates
relation
mapping
domain
range
independent variable
dependent variable

**Common Core State Standards**
A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

A point is represented on a graph using ordered pairs.

- An ordered pair \((x, y)\) is a set of numbers, or coordinates, written in the form \((x, y)\).
- The \(x\)-value, called the \(x\)-coordinate, represents the horizontal placement of the point.
- The \(y\)-value, or \(y\)-coordinate, represents the vertical placement of the point.

A set of ordered pairs is called a relation. A relation can be represented in several different ways: as an equation, in a graph, with a table, or with a mapping.

A mapping illustrates how each element of the domain is paired with an element in the range. The set of the first numbers of the ordered pairs is the domain. The set of second numbers of the ordered pairs is the range of the relation. This mapping represents the ordered pairs \((-2, 4), (-1, 4), (0, 6), (1, 8), \) and \((2, 8)\).

**Study Tip**
Sensory-Making: Each representation of the same relation serves a different purpose. Graphing the points can show the pattern between the points. A mapping shows you at a glance if elements are paired with the same element.

**Ordered Pairs**

<table>
<thead>
<tr>
<th>Table</th>
<th>Graph</th>
<th>Mapping</th>
</tr>
</thead>
<tbody>
<tr>
<td>((1, -2))</td>
<td>((0, -3))</td>
<td>((-3, 0))</td>
</tr>
<tr>
<td>((-2, 4))</td>
<td>((1, 8))</td>
<td>((8, 1))</td>
</tr>
</tbody>
</table>

The \(x\)-values of a relation are members of the domain and the \(y\)-values of a relation are members of the range. In the relation above, the domain is \((-2, 1, 0)\) and the range is \((-3, 2, 4)\).

**Example 1**
Representations of a Relation

a. Express \((2, 5), (-2, 3), (5, -2), (-1, -2)\) as a table, a graph, and a mapping.

**Table**

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>-2</td>
<td>3</td>
</tr>
<tr>
<td>5</td>
<td>-2</td>
</tr>
<tr>
<td>-1</td>
<td>-2</td>
</tr>
</tbody>
</table>

**Graph**

Graph each ordered pair on a coordinate plane.

**Mapping**

List the \(x\)-values in the domain and the \(y\)-values in the range. Draw arrows from the \(x\)-values in the domain to the corresponding \(y\)-values in the range.

b. Determine the domain and the range of the relation.

The domain of the relation is \([2, -2, 5, -1]\). The range of the relation is \([5, 3, -2]\).

**Guided Practice**

1a. Express \((4, -3), (3, 2), (-4, 1), (6, -3)\) as a table, graph, and mapping.

1b. Determine the domain and range.
In a relation, the value of the variable that determines the output is called the **independent variable**. The variable with a value that is dependent on the value of the independent variable is called the **dependent variable**. The domain contains values of the independent variable. The range contains the values of the dependent variable.

### Real-World Example 2: Independent and Dependent Variables

**Identify the independent and dependent variables for each relation.**

a. **DANCE** The dance committee is selling tickets to the Fall Ball. The more tickets that they sell, the greater the amount of money they can spend for decorations.

   - The number of tickets sold is the independent variable because it is unaffected by the money spent on decorations. The money spent on decorations is the dependent variable because it depends on the number of tickets sold.

b. **MOVIES** Generally, the average price of going to the movies has steadily increased over time.

   - Time is the independent variable because it is unaffected by the cost of attending the movies. The price of going to the movies is the dependent variable because it is affected by time.

### Guided Practice

Identify the independent and dependent variables for each relation.

2A. The air pressure inside a tire increases with the temperature.

2B. As the amount of rain decreases, so does the water level of the river.

### Graphs of a Relation

A relation can be graphed without a scale on either axis. These graphs can be interpreted by analyzing their shape.

### Example 1: Analyze Graphs

The graph represents the distance Francesca has ridden on her bike. Describe what happens in the graph.

As time increases, the distance increases until the graph becomes a horizontal line.

So, time is increasing but the distance remains constant.

At this section Francesca stopped. Then she continued to ride her bike.

### Guided Practice

Describe what is happening in each graph.

3A. **Driving to School**

3B. **Change in Income**

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**Check Your Understanding**

### Example 1

Express each relation as a table, a graph, and a mapping. Then determine the domain and range.

1. \(\{(4, 3), (-2, 2), (5, -6)\}\)

2. \(\{(-5, -7), (-1, 4), (0, -5), (-2, 3)\}\)

### Example 2

Identify the independent and dependent variables for each relation.

3. Increasing the temperature of a compound inside a sealed container increases the pressure inside a sealed container.

4. Mike's cell phone is part of a family plan. If he uses more minutes than his share, then there are fewer minutes available for the rest of his family.

5. Julian is buying concert tickets for himself and his friends. The more concert tickets he buys the greater the cost.

6. A store is having a sale over Labor Day weekend. The more purchases, the greater the profits.

### Example 3

**MODELING** Describe what is happening in each graph.

7. The graph represents the distance the track team runs during a practice. The graph represents revenues generated through an online store.

### Practice and Problem Solving

### Example 1

Express each relation as a table, a graph, and a mapping. Then determine the domain and range.

9. \(\{(0, 0), (-3, 2), (6, 4), (-1, 1)\}\)

10. \(\{(5, 2), (5, 6), (3, -2), (0, -2)\}\)

11. \(\{(-6, 1), (4, -5), (3, 2), (-1, -3)\}\)

12. \(\{(-1, 3), (3, -6), (-1, -8), (-3, -7)\}\)

13. \(\{(6, 7), (3, -2), (8, 8), (-6, 2), (2, -6)\}\)

14. \(\{(4, -3), (1, 3), (7, -2), (2, -2), (1, 5)\}\)

### Example 2

Identify the independent and dependent variables for each relation.

15. The Spanish classes are having a fiesta lunch. Each student that attends is to bring a Spanish side dish or dessert. The more students that attend, the more food there will be.

16. The faster you drive your car, the longer it will take to come to a complete stop.

### Example 3

**MODELING** Describe what is happening in each graph.

17. The graph represents the height of a bungee jumper. The graph represents the sales of lawn mowers.

18. The graph represents revenues generated through an online store.
19. The graph represents the value of a rare baseball card.

20. The graph represents the distance covered on an extended car ride.

For Exercises 21–23, use the graph at the right.

21. Name the ordered pair at point A and explain what it represents.

22. Name the ordered pair at point B and explain what it represents.

23. Identify the independent and dependent variables for the relation.

For Exercises 24–26, use the graph at the right.

24. Name the ordered pair at point C and explain what it represents.

25. Name the ordered pair at point D and explain what it represents.

26. Identify the independent and dependent variables.

Express each relation as a set of ordered pairs. Describe the domain and range.

27. Number of Fish | Total Cost
   1       | $2.50
   2       | $4.50
   5       | $10.50
   8       | $16.50

28. Express the relation in each table, mapping, or graph as a set of ordered pairs.

29. \( x \) | \( y \)
   4       | -1
   6       | 3
   -2      | 5
   7       | -3

30. Domain: \(-5, 6, 8, 9\)  
   Range: \(-6, 9\)

32. SPORTS In a triathlon, athletes swim 2.4 miles, bicycle 112 miles, and run 26.2 miles. Their total time includes transition time from one activity to the next. Which graph best represents a participant in a triathlon? Explain.

Graph A
Graph B
Graph C

Draw a graph to represent each situation.

33. ANTIQUES A grandfather clock that is over 100 years old has increased in value rapidly from when it was first purchased.

34. CAR A car depreciates in value. The value decreases quickly in the first few years.

35. REAL ESTATE A house typically increases in value over time.

36. EXERCISE An athlete alternates between running and walking during a workout.

37. PHYSIOLOGY A typical adult has about 2 pounds of water for every 3 pounds of body weight. This can be represented by the equation \( w = \frac{2}{3} b \), where \( w \) is the weight of water in pounds and \( b \) is the body weight in pounds.

a. Make a table to show the relation between body and water weight for people weighing 100, 105, 110, 115, 120, 125, and 130 pounds. Round to the nearest tenth if necessary.

b. What are the independent and dependent variables?

c. State the domain and range, and then graph the relation.

d. Reverse the independent and dependent variables. Graph this relation. Explain what the graph indicates in this circumstance.

H.O.T. Problems Use Higher-Order Thinking Skills

38. OPEN ENDED Describe a real-life situation that can be represented using a relation and discuss how one of the quantities in the relation depends on the other. Then represent the relation in three different ways.

39. CHALLENGE Describe a real-world situation where it is reasonable to have a negative number included in the domain or range.

40. CRITICAL THINKING Compare and contrast dependent and independent variables.

41. CHALLENGE The table presents a relation. Graph the ordered pairs. Then reverse the \( y \)-coordinate and the \( x \)-coordinate in each ordered pair. Graph these ordered pairs on the same coordinate plane. Graph the line \( y = x \). Describe the relationship between the two sets of ordered pairs.

42. WRITING IN MATH Use the data about the pressure of water on page 40 to explain the difference between dependent and independent variables.
**Standardized Test Practice**

43. A school’s cafeteria employees surveyed 250 students asking what beverage they drank with lunch. They used the data to create the table below.

<table>
<thead>
<tr>
<th>Beverage</th>
<th>Number of Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>milk</td>
<td>30</td>
</tr>
<tr>
<td>chocolate milk</td>
<td>112</td>
</tr>
<tr>
<td>juice</td>
<td>75</td>
</tr>
<tr>
<td>water</td>
<td>25</td>
</tr>
</tbody>
</table>

What percent of the students surveyed preferred drinking juice with lunch?

A. 25%  
B. 30%  
C. 35%  
D. 40%

44. Which of the following is equivalent to $6(5 - g) + 2(1 - g)$?

F. $2(20 - g)$  
H. $8(5 - g)$  
J. $40 - g$

45. **SHORT RESPONSE** Grant and Hector want to build a clubhouse at the midpoint between their houses. If Grant’s house is at point G and Hector’s house is at point H, what will be the coordinates of the clubhouse?

46. If $36 = 2b$, which of the following is true?

A. $b = 0$  
B. $b = 3$  
C. $b = 1$  
D. $b = \frac{3}{2}$

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**New Vocabulary**

function  
discrete function  
continuous function  
vertical line test  
function notation  
nonlinear function

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**Spiral Review**

Solve each equation. (Lesson 1-5)

47. $66 + 5 = 42$  
48. $98 = k + 11$  
49. $17 = \frac{45}{2} + 2$

50. **HOT- AIR BALLOON** A hot-air balloon owner charges $150 for a one-hour ride. If he gave 6 rides on Saturday and 5 rides on Sunday, write and evaluate an expression to describe his total income for the weekend. (Lesson 1-6)

51. **LOLLIPOPS** A bag of lollipops contains 19 cherry, 13 grape, 8 sour apple, 15 strawberry, and 9 orange flavored lollipops. What is the probability of drawing a sour apple flavored lollipop? (Lesson 0-7)

Find the perimeter of each figure. (Lesson 0-7)

52.  
53.  
54.

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**Skills Review**

Evaluate each expression.

55. $8^2$  
56. $(-6)^2$  
57. $(2.5)^2$  
58. $(-1.8)^2$  
59. $(3 + 4)^2$  
60. $(1 - 4)^2$