Review Lecture – Announcements

1. The Final Exam is on December 19th at 9 am in Science Center Hall B.
2. Structure of final similar to the midterms - Multiple Choice and Free Response.
3. Individual Exam is 3 hrs. No group portion for the final exam.
4. While the exam will cover everything, from FANCLAN to Statics to Springs, it will be slightly weighted toward module 9 and 10. Meaning you'll definitely see problems from those modules.
5. You are allowed to bring 3 equation sheets - handwritten notes on the front and back
6. Lots of Office Hours and other resources will be posted on Canvas!
Dealing with one or a few objects

"Before-and-after": Describe the change that takes place in a system, without details.
- Change in Momentum
  - Isolated system
  - Non-isolated system
- Change in Energy
  - Isolated system
  - Non-isolated system

Detailed Dynamics: Can describe all the "details" about the system.
- Static systems
- Motion at constant velocity
- Motion with constant acceleration
- Uniform circular motion (constant angular speed)
- Accelerated circular motion (constant angular accel.)
- General rotation (vector quantities)
- Harmonic oscillation

Initially
Finally

at any time
Momentum

- Momentum
  \[ \vec{p} = m \vec{v} = m \frac{d\vec{r}}{dt} \]

- Center of Mass
  \[ \vec{r}_{CM} = \frac{1}{M_{tot}} (m_1 \vec{r}_1 + m_2 \vec{r}_2 + \ldots) \]

\[ \vec{p}_{tot} = (\vec{p}_1 + \vec{p}_2 + \ldots) = M_{tot} \vec{v}_{CM} \]

\[ \vec{v}_{CM} \equiv \frac{d\vec{r}_{CM}}{dt} \]

Before-and-After: Momentum

- Isolated System:
  \[ \Delta \vec{p}_{tot} = 0 \]
  \[ \vec{p}_i = \vec{p}_f \]

  - Space
  - 2D on ice (functionally isolated)
  - Collision

- Non-Isolated System:
  \[ \Delta \vec{p}_{tot} = \int \vec{F}_{ext} dt \]
  \[ \Delta \vec{p}_{tot} = \langle \vec{F}_{ext} \rangle \Delta t \]

  - In net small force
Energy 3 types

- **Kinetic Energy:**
  \[ K = \frac{1}{2} mv^2 \]

- **Potential Energy:**
  \[ U_{grav} = mgh \]
  \[ U_{elastic} = \frac{1}{2} k(x - x_{eq})^2 \]

- **Related to net work:**
  \[ \Delta K_{cm} = \int \vec{F_{net}} \cdot d\vec{r} = \vec{F_{ret}} \cdot \Delta \vec{r} \]

- **Internal Energy:**
  - chemical, thermal, nuclear, etc.

- **Related to work done by a conservative force:**
  \[ W_{by \ grav} = -\Delta U_{grav} \]
  \[ W_{by \ spring} = -\Delta U_{elastic} \ etc. \]

Before-and-After: Energy

- **Isolated System**
  - In general:
    \[ \Delta K + \Delta U + \Delta E_{int} = 0 \]
  - **Climbing Stair**
  - **Kinesin**
  - **Conservative forces only:**
    \[ \Delta K + \Delta U = 0 \]
    **Roller Coaster**

- **Non-Isolated System**
  \[ \Delta K_{cm} + \Delta U = W_{by \ F_{NC}} = \int \vec{F_{NC}} \cdot d\vec{r} \]
  \[ \Delta K + \Delta U = W_{drag} \]
Detailed Dynamics: Statics

- **Physical Situation:**
  - Object is at rest.

- **Mathematical Description:**
  \[
  \begin{align*}
  \sum \vec{F} &= 0 \quad \sum F_x &= 0 \\
  \sum \tau &= 0 \quad \sum F_y &= 0 \\
  \tau &= RF \sin \theta = R_{\perp} F = RF_{\perp} \\
  \sin(\theta) &= \sin(180 - \theta)
  \end{align*}
  \]

Detailed Dynamics: Motion at Constant Velocity

- **Physical Situation:**
  - Object is moving at constant velocity. Net force is zero.

- **Mathematical Description:**
  \[
  \vec{v} = \frac{d\vec{r}}{dt} = \text{const.}
  \]

- **Example in x-dimension:**
  \[
  x = x_0 + v_x t
  \]
Detailed Dynamics: Motion with Constant Acceleration

Physical Situation:
- Object is moving with constant acceleration. Net force is constant:
  \[ \sum F = \text{constant} = m \ddot{a} \]
- Often: constant force such as gravity or kinetic friction:
  \[ F_{\text{grav}} = mg \]
  \[ F_{\text{friction}} = \mu_k N \]

Mathematical Description:
\[ \ddot{a} = \frac{d\ddot{v}}{dt} = \frac{d^2 r}{dt^2} = \text{const}. \]
- Example in y-dimension:
  \[ v_{f,y} = v_{i,y} + a_y t \]
  \[ y_f = y_i + v_{i,y} t + \frac{1}{2} a_y t^2 \]
  \[ 2a_y (y_f - y_i) = v_{f,y}^2 - v_{i,y}^2 \]
  \[ a_y = -g \]
  \[ a_x = 0 \]

Detailed Dynamics: Uniform Circular Motion (constant angular speed)

Physical Situation:
- Object is moving in a circle at a constant speed.
- Net inward force is required to keep object moving in circle. Examples of common forces:
  - Tension
  - Normal force
  - Static friction
  - More than one

Mathematical Description:
- Speed of rotation:
  \[ \omega = \frac{d\theta}{dt} \quad v = \omega R \quad \theta = \theta_0 + \omega t \]
  \[ v_{\text{circ}} \]
- Frequency and period:
  \[ \omega = 2\pi f \quad T = \frac{1}{f} \]
- Centripetal acceleration:
  \[ a_c = \frac{v^2}{R} = \omega^2 R \]
  \[ \text{always valid} \]
Detailed Dynamics: Still have $\alpha = \frac{N^2}{R}$

Non-Uniform Circular Motion (constant angular accel.)

- **Physical Situation:**
  - Object is moving in a circle (or rotating) with constant angular acceleration. Net torque is constant:
  $$\sum \tau = \text{constant} = I\alpha$$

- **Mathematical Description:**
  $$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} = \text{Const.}$$

- **example:**
  $$\omega_f = \omega_i + \alpha t$$
  $$\theta_f = \theta_i + \omega_i t + \frac{1}{2} \alpha t^2$$
  $$2\alpha (\theta_f - \theta_i) = \omega_f^2 - \omega_i^2$$

 Kinematics Similar to Situation

---

Detailed Dynamics: General Rotation

- **Angular velocity is a vector!**
  $$|\omega| = \text{speed}$$
  Direction $\Rightarrow RHR$

- **Torque is a vector:**
  $$\vec{\tau} = \vec{R} \times \vec{F}$$

  $RHR \pm \frac{\pi}{2}$

- **Angular momentum:**
  $$\vec{L} = I\vec{\omega}$$
  Helpful when $\vec{\tau} = 0$

- **Newton’s 2nd for rotation:**
  $$\sum \vec{\tau} = \frac{d\vec{L}}{dt}$$

- **Angular frequency of precession:**
  $$\omega_{\text{precess}} = \frac{\tau}{L \sin \phi}$$

sense of rotation of the wheel

The right hand rule for angular quantities.

$\bullet$ (out of page)

$\times$ (into page)

$\vec{R}$

$\vec{F}$

$\vec{L}$

$\vec{\omega}$

$\vec{\tau}$

$\phi$
Detailed Dynamics: Simple Harmonic Motion

- **Physical Situation:**
  - Object has linear restoring force (like a spring).
  - Approximate behavior of all systems near a point of stable equilibrium:

- **Mathematical Description:**
  - Mass on a spring:
    \[ x(t) = A \cos(\omega t) \]
    or
    \[ x(t) = A \sin(\omega t) \]
  - \( \omega = \sqrt{\frac{k}{m}} \)

Dealing with Many, Many Particles

- **Fluids:** Collective motion of many particles
  - Statics
    - Pressure
    - Buoyancy
    - Surface tension
  - Dynamics
    - Continuity
    - High Reynolds Number: neglect viscosity
    - Low Reynolds Number: neglect inertia

- **Statistical Physics**
  - "Statics" = equilibrium
  - Thermal kinetic energy
  - Many particles at equilibrium: Boltzmann Distribution
  - Dynamics
    - Average motion of one particle: Random Walk
    - Average motion of many particles: Diffusion
Fluid Statics

- Pressure: \[ P = \frac{\text{force}}{\text{area}} \]
- Incompressible fluids (liquids):
  \[ \Delta P = \rho g \Delta h \]
- Compressible fluids (gases):
  \[ PV = nRT = Nk_BT \]
  \[ \frac{P}{P_0} = e^{-\frac{mgh}{k_BT}} \]

Buoyancy:

\[ F_{\text{buoyant}} = (\rho_{\text{fluid}})(V_{\text{displaced}})g \]

Surface tension:

\[ \gamma = \frac{\text{force}}{\text{length}} \]

- For curved surface:
  \[ P_{\text{in}} - P_{\text{out}} = \frac{2\gamma}{R} \]

Fluid Dynamics

- Continuity:
  \[ Q_1 = Q_2 \quad A_1v_1 = A_2v_2 \]

Reynolds Number:

\[ Re = \frac{\rho v l}{\eta} \]

- High Reynolds Number (neglect viscosity):
  \[ F_{\text{drag}} = \frac{1}{2} C_D \rho A v^2 \]
- Bernoulli:
  \[ P + \frac{1}{2} \rho v^2 + \rho g y = \text{constant} \]

- Low Reynolds Number (neglect inertia):
  \[ F_{\text{drag}} = 6\pi \eta R v \]

Poiseuille:

\[ Q = \frac{\pi R^4 \Delta P}{8\eta L} \]
Statistical Physics: Equilibrium

- Many particles at thermal equilibrium:
  - Thermal kinetic energy:
    - One-dimensional motion
      \[ \left\langle \frac{1}{2} m v_x^2 \right\rangle = \frac{1}{2} k_B T \]
    - Three-dimensional motion
      \[ \left\langle \frac{1}{2} m v^2 \right\rangle = \frac{3}{2} k_B T \]
      \[ \langle K \rangle = \frac{3}{2} k_B T \]
  - Boltzmann distribution:
    \[ \frac{\text{Prob}(2)}{\text{Prob}(1)} = e^{\frac{(U_2 - U_1)}{k_B T}} \]
    \[ \frac{P}{P_0} = e^{\frac{mgh}{k_B T}} \]
    \[ U_1 > U_2 \text{ more at lower energy state} \]

Statistical Physics: Motion

- Average motion of one particle: Random Walk (1D)
  - N steps: length: \( \delta \) time: \( \tau \)
    \[ \left\langle x \right\rangle = 0 \quad \left\langle x^2 \right\rangle = N \delta^2 = \frac{t}{\tau} \delta^2 \]
  - Relationship to macroscopic parameters:
    \[ D = \frac{\delta^2}{2\tau} \quad \left\langle x^2 \right\rangle = 2D \tau \]
    \[ D = \frac{kT}{f} \quad \left\langle x^2 \right\rangle = 2D \tau \]
    \[ \text{given by experiment} \]
    \[ D = \frac{kt}{f} \quad \left\langle x^2 \right\rangle = 2D \tau \]
    \[ \text{2D} \quad \left\langle x^2 \right\rangle = 4D \tau \]
    \[ \text{3D} \quad \left\langle x^2 \right\rangle = 6D \tau \]

- Average motion of many particles: Diffusion (1D)
  - Flux:
    \[ J_x = \frac{\text{molecules}}{\text{area} \cdot \text{time}} = -D \frac{dc}{dx} \]
  - Diffusion equation ("smooths out bumps"):
    \[ \frac{dc}{dt} = D \frac{d^2 c}{dx^2} \]
Demo: Bed of Nails

A very careless person is going to lie on a bed of nails.

The bed has approximately 1,000 (1 mm diameter) nails placed in it so that they extend through the board. The nails are spaced about 2 cm apart. If the person has a weight of 600 N and skin has ultimate tensile strength of $\sigma = 27.2 \times 10^6$ Pa, what will happen and why? How would this change (if at all) if he lays down on a single nail with a 1 mm diameter?

a) He will be severely punctured (needles will break completely through his skin into his organ), all for the sake of Physics.

b) He will be slightly punctured (needles will just barely break through his skin).

c) He will be unharmed since the needles will not break through his skin.