Physical Sciences 2
Exam 2 (Group)
Thursday, November 13, 2018

Group Member Name(s): Key

Section TF(s): 

Do not turn the page until you are told to begin. You will be given 45 minutes to complete this exam. Show all your work on the exam itself; no credit will be given for anything written on other paper. Please box your final answer to each calculation.

You may use a calculator if you have brought one. You may refer to two 8.5”x11” sheet of notes, which must be in your own handwriting. Turn in your notes along with the exam when time is called.

This exam contains 5 sheets of paper (including this one), with 5 problems.

Do not write in the following table; it will be used for grading.

| Problem 1 | ___ / 30 |
| Problem 2A | ___ / 7 |
| Problem 2B | ___ / 9 |
| Problem 3 | ___ / 15 |
| Problem 4 | ___ / 13 |
| Problem 5 | ___ / 26 |
| Total | ___ / 100 |
Problem 1: Multiple Choice [30 points]

For each of the following questions, circle the letter(s) corresponding to the best answer(s) from the options given. Partial credit will be given when specified and for those problems asking you to “Circle all that apply.”

a) [5 pts] Suppose you start an antique car by exerting a constant force of 400 N on its crank for 0.250 s. What total angular momentum do you impart to the engine if the handle of the crank is 0.5 m from the pivot and the force is exerted so as to create maximum torque for the entire time? (You may neglect friction; choose the closest answer.)

(A) 50 kg · m²/s
B) 100 kg · m²/s
C) 200 kg · m²/s
D) 800 kg · m²/s
E) 1600 kg · m²/s

\[ 0.5 \times 400 \text{N} \times 0.250 \text{s} = 50 \text{ kg} \cdot \text{m}^2/\text{s} \]

b) [5 pts] You suspend a weight from a 20-cm-long wire; the wire stretches by 1 cm. You cut the wire in half and suspend the same weight from the two 10-cm pieces of wire. How much does each 10-cm piece of wire stretch?

(a) 0.25 cm
b) 0.5 cm
c) 1 cm
d) 2 cm
Physical Sciences 2

c) [5 pts] Suppose you drop a ball from a high tower and it falls freely under the influence of the gravitational force. Ignore air drag. Which of the following statements is true? **Circle all that apply.**

a) The kinetic energy of the ball increases by equal amounts in equal times.

b) The kinetic energy of the ball increases by equal amounts over equal distances.

c) There is zero work done on the ball by the gravitational force as it falls.

d) The work done on the ball by the gravitational force is negative as it falls.

e) The total mechanical energy of the ball decreases as it falls.

As it falls: \[ P.E. \rightarrow K.E. \]

\[ mgh \rightarrow \frac{1}{2} mv^2 \]

For each \( \Delta h \), \( mgh \Delta h \) is converted to kinetic energy.

d) [5 pts] How high up a mountain, whose base is at sea level \( (P = 1 \text{ atm}) \), would you have to climb for the pressure to drop to 13% of its value at the base. \( (H \text{ is the scale height}) \)

A) \( H/2 \)

B) \( H \)

C) \( 2H \) \( \square \)

D) \( e^{H/2} \)

E) \( e^{-H/2} \)

\[ P_t = P_0 e^{-\frac{y}{H}} \]

If \( y = 2H \)

\[ P_t = P_0 e^{-2} = 0.13 P_0 \]

13%
e) [10 pts] A container is filled with a liquid having the properties: $\gamma_{\text{liquid}} = 0.5 \, \text{N/m}$, $\rho_{\text{liquid}} = 100 \, \text{kg/m}^3$. Five points are labeled in the diagram: points C and D are inside small air bubbles. The radius of the bubble around C is $R_C = 1 \, \text{mm}$, and the radius of the bubble around D is $R_D = 5 \, \text{mm}$. At which point is the pressure highest? Take the gravitational acceleration to be $g = 10 \, \text{m/s}^2$ for this problem.

You will be given PARTIAL CREDIT for your work on this problem, so please show your work below.

A) Point A
B) Point B
C) Point C
D) Point D
E) Point E
F) Points C and D (which are the same)
G) Points C, D, and E (which are the same)
H) Points B, C, D, and E (which are the same)

\[ \rho_{\text{liquid}} = 100 \, \text{kg/m}^3 \]

\[ \text{20 cm} \]

\[ \text{1 m} \]

\[ \text{Comparing Pressures: B} > \text{A bc it's deeper} \]
\[ \text{C} > \text{B bc C is in bubble} \]
\[ \text{D} > \text{E bc D is in bubble} \]

So is \( C > D, C < D \), or \( C = D \)?

\[ \frac{P_C - P_D}{(1 \, \text{atm} + \rho g (20 \, \text{cm}) + \frac{2 \gamma}{R_C}) - (1 \, \text{atm} + \rho g (20 \, \text{cm}) + \rho g (80 \, \text{cm}) + \frac{2 \gamma}{R_D})} \]

\[ = \frac{2 \gamma}{R_C} - \rho g (0.8 \, \text{m}) - \frac{2 \gamma}{R_D} \]

\[ = \frac{2 \cdot 0.5 \, \text{Pa}}{10^{-3}} - 100 \cdot 10^{-3} \cdot 0.8 \, \text{Pa} - \frac{2 \cdot 0.5}{5 \times 10^{-3}} \]

\[ = 0 \, \text{Pa} \]

So \( P_C = P_D \)
Problem 2A: Stick... [7 pts]

The figure to the right shows a force $F$ (15 N) applied in several different ways to a 5.0 m long rod with a pivot at one end (point P).

Rank the torque due to the applied force in each case from least to greatest.

Use the convention that forces tending to induce counterclockwise motion yield positive torque, and forces tending to induce clockwise motion yield negative torque.

Express your answer in the format: $3 < 2 < 4 < 5 = 1$ (for example). Where the smallest torque is on the left, greatest torque is on the right, and either "<" or "=" is between each.

$4 < 5 < 2 < 3 < 1$

Problem 2B: Stick with it! [9 pts]

One end of a stick with mass $m$ and length $l$ is pivoted on a wall, and the other end rests on a frictionless floor, as shown in the figure. Let $F_L$ and $F_R$ be the vertical components of the forces acting on the left end of the stick and on the right end at the pivot, respectively. Find an expression for $F_R$ and $F_L$ in terms of $m$, $l$, $\theta$, and any relevant physical constants.

If pivot at $F_R$ then

$\sum \tau = 0$

$F_g \frac{l}{2} \sin(90+\theta) + F_L \frac{l}{2} \sin(90+\theta) = 0$

$F_L = \frac{F_g}{2} = \frac{mg}{2}$

$\sum F_y = 0$

$F_L + F_R - F_g = 0 \Rightarrow F_R = mg + \frac{mg}{2} - mg = 0 \Rightarrow F_R = \frac{mg}{2}$
Problem 3: Don’t get wet! [15 pts]

A pontoon bridge floats on 10 pontoons. Each cylindrical pontoon has a mass \( m = 20 \text{ kg} \), a diameter \( d = 20 \text{ cm} \), and a length \( L = 2 \text{ meters} \). Calculate the maximum number of people that the bridge can support without the pontoons becoming fully submerged. Assume each person has a mass \( M = 60 \text{ kg} \), and the bridge is in fresh water with a density \( \rho = 1000 \text{ kg/m}^3 \). (You may neglect the mass of the wooden bridge deck.)

\[
AF_B = 0
\]
\[
\Rightarrow F_B = F_g
\]
\[
\Rightarrow V_{\text{disp}} \cdot g = (N \cdot m_{\text{person}} + 10 \cdot m_{\text{pontoon}}) \cdot g
\]
\[
= 10 \cdot \frac{1000 \text{ kg}}{1 \text{ m}^3} \cdot \frac{60 \text{ kg}}{10 \text{ m}^3} \cdot \frac{1 \text{ m}}{1 \text{ s}^2}
\]
\[
= 0.63 \text{ m}^3
\]
\[
N = \frac{630 - 10 \cdot 20}{60} = 7.17
\]

7 people
Physical Sciences 2

Problem 4: Roll away [13 pts]

A cylinder is released from rest and rolls without slipping toward the bottom of a circular-shaped ramp. The cylinder has a moment of inertia $I_A = \frac{1}{2} m_A R_A^2$. Recall that for rolling without slipping, $v_{\text{CM}} = \omega R$.

a) Calculate the speed of the cylinder’s center of mass at the moment the cylinder reaches Point O. Hint: what kinds of energy does a rolling object have?

\[ mgh = \frac{1}{2} m v_{\text{CM}}^2 + \frac{1}{2} I \omega^2 \]

\[ \Rightarrow mgh = \frac{1}{2} m v_{\text{CM}}^2 + \frac{1}{2} m R_A^2 \frac{v_{\text{CM}}^2}{R_A^2} \]

\[ g h = v_{\text{CM}}^2 \left( \frac{1}{2} + \frac{1}{4} \right) \]

\[ v_{\text{CM}} = \sqrt{4.75 g h} = 5.11 \text{ m/s} \]

\[ 9.18 \text{ (m/s)}^2 \]

b) As the cylinder rolls down the ramp, determine the sign of the work done on the center of mass by each of the following forces: (circle the best answer in each case; recall that the cylinders are rolling without slipping)

The work done on the cylinder by normal force from the ramp: 
- negative
- zero
- positive

The work done on the cylinder by gravitational force on the cylinder:
- negative
- zero
- positive

The work done on the cylinder by kinetic friction force from the ramp:
- negative
- zero
- positive

The work done on the cylinder by static friction force from the ramp:
- negative
- zero
- positive

Page 7 of 9
Problem 5: Look out below! [26 points]

During a test of a new bungee cord, a professional bungee jumper dives from a tall cliff. The cord has an un-stretched length $L_0 = 16$ m, an effective spring constant $k = 1 \times 10^4$ N/m, and is very lightweight.

Consider three different times of the jump as shown in the diagram (**not to scale**):

**Time $t_1$:** The cord is un-stretched (at its initial length $L_0$) and the jumper is at rest at the top of the cliff.

**Time $t_2$:** The jumper has fallen, stretching the cord to its maximum length $y_{\text{max}}$, and is oscillating up-and-down like a damped harmonic oscillator.

**Time $t_3$:** The jumper has finally come to rest at equilibrium with the cord at a final length $y_{\text{final}}$.

---

**a) During the oscillations (at time $t_2$), the jumper completes one and a half cycles every 0.85 seconds. Calculate the mass $m$ of the bungee jumper. You may ignore friction, drag, and other damping forces for this part of the calculation.**

*Note: A cycle is defined as one full oscillation and the time for one full oscillation is the period: $T=2\pi/\omega$.***

\[
T = \frac{1}{1.5} \times 0.85 = 0.567 \text{s}
\]

\[
\omega = \sqrt{\frac{k}{m}} = \frac{2\pi}{T} \implies \frac{k}{m} = \frac{4\pi^2}{T^2} \implies m = \frac{kT^2}{4\pi^2} = 81.34 \text{ kg}
\]

*continued on next page...*
Physical Sciences 2

Problem 5 continued

b) The bungee cord is a cylinder made of rubber-like material with a Young's modulus \( Y = 3 \times 10^7 \) Pa. Calculate the radius \( r \) of the cord. **Hint: effective spring constant is** \( k = 1 \times 10^4 \) N/m.

\[
\frac{F}{A} = \frac{Y \Delta L}{L_0} \Rightarrow F = \frac{YA}{L_0} \Delta L
\]

\[
K_{eff} = \frac{YA}{L_0}
\]

\[
= \frac{A}{Y} \Rightarrow \pi r^2 = \frac{K_{eff} \cdot L_0}{Y}
\]

\[
r = \sqrt{\frac{K_{eff} \cdot L_0}{\pi Y}}
\]

\[
y = 3 \times 10^7 \text{ Pa}
\]

\[
L_0 = 16 \text{ m}
\]

\[
K_{eff} = 10^4 \text{ N/m}
\]

\[
r = 0.0412 \text{ m}
\]

c) Find the value of the final position, \( y_{final} \), when the jumper has come to rest at equilibrium.

\[
y_{final} = L_0 + \Delta L \Rightarrow (16 + 0.08) \text{ m}
\]

\[
= 16.08 \text{ m}
\]

\[
downward
\]

\[
F_{elastic} = m g
\]

\[
K \Delta L = m g
\]

\[
\Delta L = \frac{m g}{K} = \frac{81.34 \cdot 9.81}{10^4} = 0.08 \text{ m}
\]

\[
K = 10^4 \text{ N/m}
\]

\[
y = 3 \times 10^7 \text{ Pa}
\]

\[
L_0 = 16 \text{ m}
\]

\[
K_{eff} = 10^4 \text{ N/m}
\]

\[
r = 0.0412 \text{ m}
\]

d) Calculate the **total work** done by all damping forces (e.g. friction and air drag) between the start of the jump at time \( t_1 \) and the final equilibrium at time \( t_3 \). **Hint: you don’t have to do any integrals.**

\[
W_{nc} = \Delta E_{mech} = \Delta U + AK
\]

\[
= \Delta U_g + \Delta U_{elastic}
\]

\[
= mg y_f + \frac{1}{2} K (\Delta L)^2 = 12.85
\]

\[
81.34 \text{ kg}
\]

\[
16.08 \text{ m}
\]

\[
10^4 \text{ N/m}
\]

\[
0.08 \text{ m}
\]