Physical Sciences 2: Assignments for Nov 15 – 29, 2018
Homework #9: High and Low Reynolds Number Flow, Statistical Mechanics
Due Thursday, Nov. 29 at 9:00AM

This assignment must be turned in by 9:00AM on Thursday, November 29. Late homework will not be accepted. Please write your answers to these questions on a separate sheet of paper with your name and your section TF’s name written at the top. Turn in your homework to your section TA's box.

You are encouraged to work with your classmates on these assignments, but please write the names of all your study group members on your homework.

After completing this homework, you should…

- Be able to quantitatively and qualitatively describe the meaning of the Reynolds number
- Understand the differences between high Re flow and low Re flow
- Be able to explain the concept of dynamical similarity
- Understand the derivation of the continuity equation
- Know the derivation of and how to use Bernoulli’s equation
- Be able to qualitatively and quantitatively explain the meaning of viscosity
- Be able to use Newton’s law of viscosity
- Understand the properties of low Re number flows
- Be able to use the Stokes equation
- Be able to use the Poiseuille equation
- Know the meaning of Brownian motion
- Know how statistical mechanics is used to describe pressure and temperature
- Be able to calculate the average kinetic energy and average velocity of gas molecules
Reynolds Number

- The Reynolds number $RE$ is a number used to
  - determine whether a flow is laminar ($RE < 2000$) or turbulent ($RE > 2000$)
  - determine whether viscous drag ($RE << 1$) or pressure drag ($RE >> 1$) dominates
    \[ F_{\text{drag}} = 6\pi \eta RV \]
    \[ F_{\text{drag}} = \frac{1}{2} C_d JAV^2 \]
  - determine the characteristics of different types of flow (high $RE$ flow vs. low $RE$ flow)
    \[ RE = \frac{\rho v l}{\eta} \]

High $RE$ Flow

- $RE = \frac{\rho v l}{\eta}$ large; could be a large object moving at a normal speed in a fluid of normal viscosity
- Bernoulli Equation
  - relates pressure, height, and velocity at two points in a high Reynolds flow
    \[ P_1 + \rho g h_1 + \frac{1}{2} \rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2} \rho v_2^2 \]

- Venturi Effect
  - relates pressure and velocity in a high Reynolds flow
    \[ P_1 - P_2 = \frac{1}{2} \rho \left( v_2^2 - v_1^2 \right) \]
  - lower pressure, higher velocity in the narrow portion of tube
Low RE Flow

- RE = \frac{\rho v L}{\eta}

Small objects moving slowly through a fluid with any viscosity could be a large object moving at low speed through a fluid with a high viscosity.

Stoke's Equation

- Drag force acting on an object moving through a low RE number fluid

\[ F_{\text{drag}} = 6\pi \eta v R \]

\eta - viscosity
v - velocity
R - radius

Poiseuille Equation

- Relate the flow rate to the pressure change and viscosity

\[ Q = \frac{\pi}{8} \frac{\Delta P}{\eta L} R^4 \]

Q - flow rate
\Delta P - pressure difference: \Delta P = P_{\text{high}} - P_{\text{low}}
\eta - viscosity
L - length of pipe
R - radius of pipe

Statistical Mechanics

- Cannot track every single molecule; instead only consider averages

- The average kinetic energy of a molecule is

\[ \langle KE \rangle = \frac{3}{2} k_B T \]

\[ k_B = 1.38 \times 10^{-23} \text{ J/K} \] Boltzmann constant
T - Temperature (in Kelvin)

- The RMS (root-mean-square) velocity of a molecule is

\[ \langle KE \rangle = \frac{1}{2} m \langle v^2 \rangle = \frac{3}{2} k_B T \]

\[ V_{\text{rms}} = \sqrt{\langle v^2 \rangle} = \sqrt{\frac{3k_B T}{m}} = \sqrt{\frac{2\langle KE \rangle}{m}} \]

- This is the speed of a molecule that has the average kinetic energy; \( V_{\text{rms}} \) is very close to the average speed of a typical molecule.
0. Reflections on Last Assignment (1 pt)

Pick one question from Homework 8 that you found particularly difficult and

a) describe any mistakes or misunderstandings you made

b) describe the best strategies to ensure you learn from your mistakes and won’t have the same misunderstanding again

1. Squeeze-It! (1 pt)

The schematic at right shows a squirt bottle of the sort demonstrated in class (and used in many chemistry labs). When you squeeze the body of the bottle, you increase the pressure inside the bottle slightly. Given the approximate pressures and dimensions shown, what is the speed of the water as it emerges from the nozzle? (You may assume that the cross-sectional area of the bottle is much larger than the area of the nozzle.)

2. Clogging your artery (2 pts)

Suppose that blood flows through a large, unobstructed artery with a speed of 0.5 m/s. Assume the artery is horizontal.

a) If part of the artery becomes obstructed, and the obstructed region has a radius 1/3 that of the unobstructed region, by how much does the blood pressure drop in the obstructed region?

b) Under what conditions could that drop in pressure cause the artery to collapse?

3. Punctured aorta! (1 pt)

During open-heart surgery, a surgeon accidentally tears a small circular hole (2 mm diameter) in the aorta. If that blood squirts straight up into the air, how high will it squirt? (Neglect air resistance.)
4. The giraffe, again (1 pt)

A giraffe’s heart must provide sufficient diastolic pressure to overcome both the hydrostatic pressure that results from the height of its head above its heart and the viscous resistance to fluid flow in the artery that travels from its head to its heart. Given the following information:

- Blood has a density of 1060 kg/m$^3$
- A giraffe’s head is about 3 meters above its heart
- The artery from the giraffe’s heart to its head has a radius of about 3 mm
- The giraffe needs to supply 9 mL of blood per second to its brain
- The viscosity of whole blood is about $4 \times 10^{-3}$ kg·m$^{-1}$·s$^{-1}$

Calculate the diastolic pressure that must be produced by the giraffe’s heart in order for the pressure in its head to be 1 atmosphere.

5. Turbulence in the bloodstream (2 pts)

For steady flow in a circular pipe, the transition between laminar and turbulent flow occurs at a Reynolds number on the order of $10^3$.

a) Estimate the Reynolds number for blood flow in the aorta and determine if the flow there is laminar, turbulent, or too close to call.

b) Do the same for flow in both veins and capillaries. (Hint: you might need to make some estimates about size and flow rate.)
6. Measuring the viscosity of blood (2 pts)

The viscosity of blood depends on the red blood cell concentration: the higher the concentration, the higher the viscosity. A not-so-recent article described a simple apparatus for measuring the viscosity of blood (*Journal of Non-Newtonian Fluid Mechanics, 94, 47-56, (2000)*). The figure at right shows a schematic of the device. There are two cylindrical “riser tubes” that are each 3 mm in diameter. Connected between these two tubes is a capillary tube with an interior diameter of 0.80 mm and a length of 100 mm. The device is filled with blood through a stopcock such that the level of blood in one riser tube is higher than the level of blood in the other. This difference in heights creates a pressure difference across the capillary tube, which causes blood to flow through the capillary tube.

The levels of blood in the two riser tubes are measured as a function of time as shown in the graph below:

![Graph showing height of blood in riser tubes over time](image)

a) Given the initial height difference of 58 mm between the levels in the two riser tubes, calculate the initial pressure difference across the capillary tube.

b) From the graph, estimate the initial slope of the curves that show the height of blood as a function of time. What is the initial volumetric flow rate (in mL/s) for this sample of blood?

c) Assuming that the viscous resistance to flow can be neglected everywhere except in the capillary tube, estimate the viscosity of this sample of blood.