Lecture 9a: High Reynolds Number Flows

Background: Laminar versus Turbulent flows

Qualitatively, fluid motion comes in two types:

- **Laminar flow**: regular flow with motion of smooth “lamina” or sheets
- **Turbulent flow**: Irregular, fluctuating, chaotic motions
  (modeling this flow is one of toughest problems in all of physics)

Every flow can be characterized as laminar or turbulent

Keywords: Streamlines and steady flow

- In a **steady flow** the measurable quantities of density, pressure, and velocity at a given position do not change with time. The velocity vector is everywhere tangent to the **streamline**.
  *(a streamline is the path followed by a tracer particle in a fluid)*

- Two types of flows: **laminar** and **turbulent**

- **Reynolds number** determines whether a flow is turbulent or laminar (transition at Re~2000), and what the streamlines look like
Brief outline of the lecture
1. Fluid motion comes in two types: Laminar and turbulent
2. The Reynolds number $Re$ determines the qualitative features of the flow
   - $Re$ can be interpreted as the ratio of pressure drag to viscous drag on an object in a flow
   - Dynamical similarity: at the same $Re$, flows have the same qualitative features
3. Bernoulli’s equation applies at high $Re$. It relates fluid pressure, speed, and gravitational potential energy

Learning objectives: After this lecture, you will be able to:

1. Qualitatively and quantitatively explain the meaning of Reynolds number $Re$.

2. Understand the qualitative differences between high Reynolds number flows and low Reynolds number flows.

3. Explain the concept of dynamical similarity and use it to design scale models.

4. Explain that conservation of mass leads to the continuity equation $A_1 v_1 = A_2 v_2$, which relates fluid velocities in any incompressible fluid.

5. Explain the connection between Bernoulli’s equation and the work-kinetic energy theorem.

6. Use Bernoulli’s equation to solve fluid flow problems.
Activity 1: Intro to Reynolds number \( Re \)

The Reynolds number is defined as 

\[
Re = \frac{\text{Inertial forces}}{\text{Viscous forces}} = \frac{\rho v l}{\eta}, \quad \text{where:}
\]

\( \rho \) = density of fluid, \( v \) = velocity of fluid (or object)  
\( \eta \) = viscosity of fluid, \( l \) = length scale (body size)

\[
\eta_{\text{water}} = 10^{-3} \text{Pa} \cdot \text{sec} \quad \eta_{\text{air}} = 1.8^{-5} \text{Pa} \cdot \text{sec} \\
\rho_{\text{water}} = 10^3 \text{kg/m}^3 \quad \rho_{\text{air}} = 1.4 \text{ kg/m}^3
\]

The Reynolds number can be expressed as the ratio of Pressure Drag and Viscous Drag:

Pressure drag: \( F_D = \frac{1}{2} C_D \rho A v^2 \) (pushing fluid out of the way)
Viscous drag: \( F_D = 6\pi \eta R v \) (sticky drag)

For \( Re \ll 1 \), viscous drag dominates  
For \( Re \gg 1 \), pressure drag dominates

1. Divide the expression for pressure drag by the expression for viscous drag (ignoring constants \& \( C_D \)). How does this compare to \( Re = \frac{\rho v l}{\eta} \)?

\[
\frac{F_{\text{pressure drag}}}{F_{\text{viscous drag}}} = \frac{\rho A v^2}{\eta R v} = \frac{\rho v (A/R)}{\eta} \text{ length that relates to cross-section}
\]

Order-of-magnitude estimates of the Reynolds number for different organisms (you will need to estimate \( v \) and \( l \)):

2. Flow of water around the body of a swimmer

\[
Re = \frac{\rho v l}{\eta} = \frac{10^3 \cdot 1 \cdot 0.1}{10^{-3}} \approx 10^5 \text{ turbulent}
\]

3. Flow around a swimming bacterium in water (Note: \( v \sim 30 \mu m/sec \))

\[
Re = \frac{\rho v l}{\eta} = \frac{10^3 \cdot 3 \times 10^{-5} \times 10^{-6}}{10^{-3}} = 3 \times 10^{-5} \text{ laminar}
\]

4. You want to design a model experiment to mimic real flow conditions, but in a 1/10-scale model of an airliner. Given that \( l \to l/10 \). How might you achieve \textbf{Dynamical Similarity} in the experiment (i.e. same Reynold number as a real airliner)

\[
Re = \frac{\rho v l}{\eta} \quad l \to l/10 \quad v \to 10v \quad 5000 \text{ mph} \quad \rho \to 10\rho
\]
Activity 2: Bernoulli eq. in terms of work & energy

If you recall from Module 4, we discussed the work kinetic energy theorem: \( W_{Total} = \Delta K \). Rewriting this gives \( W_{Total} = W_{cons} + W_{nc} = \Delta K \). Using \( W_{cons} = -\Delta U \) we get: \( W_{nc} = \Delta K + \Delta U \).

Applying this to a fluid can give the following relation between the fluid’s speed, pressure, and height: \( p_1 + \frac{1}{2} \rho v_1^2 + \rho gh_1 = p_2 + \frac{1}{2} \rho v_2^2 + \rho gh_2 \). You can derive this!

Express answer below in terms of: \( P_1, P_2, A_1, A_2, v_1, v_2, \Delta l_1, \Delta l_2, y_1, y_2, g, \) and \( \rho \)

1. Would you classify the pressure forces \( F_1 = P_1A_1 \) and \( F_2 = P_2A_2 \) as conservative or non-conservative? Explain.

2. Find the work done by the pressure forces \( F_1 = P_1A_1 \) and \( F_2 = P_2A_2 \) as the fluid is pushed a distance \( \Delta l_1 \) at point 1 and \( \Delta l_2 \) at point 2 (i.e. find \( W_{nc} \)).

\[
W_{nc} = F_1 \Delta l_1 - F_2 \Delta l_2 = P_1 A_1 \Delta l_1 - P_2 A_2 \Delta l_2
\]

3. Find the change in potential energy \( \Delta U \). Note: replace \( m_i \) by \( \rho V_i = \rho A_i \Delta l_i \)

\[
\Delta U = U_f - U_i = m_2 g y_2 - m_1 g y_1 = \rho A_2 \Delta l_2 y_2 - \rho A_1 \Delta l_1 y_1
\]

4. Find the change in kinetic energy \( \Delta K \). Note: replace \( m_i \) by \( \rho V_i = \rho A_i \Delta l_i \)

\[
\Delta K = \frac{1}{2} m_f V_f^2 - \frac{1}{2} m_i V_i^2 = \frac{1}{2} \rho A_2 \Delta l_2 V_2^2 - \frac{1}{2} \rho A_1 \Delta l_1 V_1^2
\]

5. Now you’re ready to write down \( W_{nc} = \Delta K + \Delta U \)

\[
P_1 A_1 \Delta l_1 - P_2 A_2 \Delta l_2 = \rho g A_2 \Delta l_2 y_2 - \rho g A_1 \Delta l_1 y_1 + \frac{1}{2} \rho A_2 \Delta l_2 V_2^2 - \frac{1}{2} \rho A_1 \Delta l_1 V_1^2
\]

6. Use \( A_1 \Delta l_1 = A_2 \Delta l_2 \) to simplify your result in (5). Your final result should be:

\[
p_1 + \frac{1}{2} \rho v_1^2 + \rho gh_1 = p_2 + \frac{1}{2} \rho v_2^2 + \rho gh_2
\]

and it is called the Bernoulli equation.
Activity 3: Bernoulli equation

Bernoulli's equation relates pressure, height, and velocity in high Reynolds number flow.

\[ p_1 + \frac{1}{2} \rho v_1^2 + \rho gh_1 = p_2 + \frac{1}{2} \rho v_2^2 + \rho gh_2 \]

1. Use the Bernoulli equation to find the pressure of the fluid at the bottom of the tank, at a depth \( h \).

\[ p_2 = p_1 + \rho g h_1 - \rho g h_2 \]

\[ = p_1 + \rho g (h_1 - h_2) = p_1 + \rho g h \]

2. Use the Bernoulli equation to find the velocity of the fluid as it exits the small opening at the bottom of the tank.

\[ V_1 = 0 \quad P_1 = P_3 \]

\[ \rho g h_1 = \frac{1}{2} \rho V_3^2 + \rho g h_3 \]

\[ \frac{1}{2} \rho V_3^2 = \rho g (h_1 - h_3) = \rho g H \]

\[ V_3 = \sqrt{2gH} \]

Bonus 1: Find the horizontal distance \( \Delta x \) where the flow strikes the table. (the table is at a height \( h \) below the small opening at the bottom of the tank).

\[ h = \frac{1}{2} a t^2 = \frac{1}{2} g t^2 \]

\[ t = \sqrt{\frac{2h}{g}} \]

\[ \Delta x = \text{speed} \times \text{time} = \sqrt{\frac{2h}{g}} \times \sqrt{2gH} = 2 \sqrt{hH} \]

Bonus 2: If the flow reaches a height of 12m, what was the velocity of the water exiting the Geyser?

\[ \rho g y_1 + \frac{1}{2} \rho v_1^2 = p_2 + \frac{1}{2} \rho v_2^2 \]

\[ \rho g y_1 = \frac{1}{2} \rho v_2^2 \]

\[ V_2 = \sqrt{2g y_1} \]

\[ \rho g y_1 = \frac{1}{2} \rho V_2^2 \]

\[ V_2 = \sqrt{2g y_1} \]
Activity 4: Venturi effect

Venturi effect relates the pressure and velocity in a fluid flow. A tremendous number of fluid flow applications can be explained with the Venturi effect. We will see a few demonstrations of the Venturi effect.

The continuity equation \( Q = A_1 v_1 = A_2 v_2 \) tells us the speed of a fluid increases when it enters a smaller cross-sectional area \( A \). What about pressure?

1. Use the Bernoulli equation to figure out if the pressure fluid increases or decreases when it enters a smaller cross-sectional area \( A \), in a tube (as it does in the figure to the right). Use \( h_1 = h_2 \) to simplify the equation.

\[
\frac{p_1 + \frac{1}{2} \rho v_1^2}{g y_1} = \frac{p_2 + \frac{1}{2} \rho v_2^2}{g y_2}
\]

2. In order to ventilate their burrows, Prairie dogs build burrows with ends at two different heights. Why do they have different heights? Which way does the wind flow inside the burrows?

**Demo:** Give an explanation why the beach ball is in a ‘stable’ equilibrium (when it moves from the center, a restoring force pushes it back in place).

**Demo:** The Magnus effect: Can you use the Venturi effect to explain why a baseball can curve?
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Physical Sciences 2: Lecture 9a

One-Minute Paper

Your name: ___________________________ TF: ___________________________

Names of your group members: ___________________________

• Please tell us any questions that came up for you today **during** lecture. Write “nothing” if no questions(s) came up for you in class from 9:30am–11am.

• What single topic left you **most confused** after today’s class?

• Any other comments or reflections on today’s class?