Physical Sciences 2

Lecture 8b: Gases and Fluid flow

Pre-reading:

For this lecture, you need to review the ideal gas law from Chemistry (see below)

Note: The ideal gas law you learned in chemistry is: \( PV = nRT \), where \( n \) = # moles and \( R = 8.31 \text{J/mol/K} \) is the universal gas constant.

Turns out there are many variations of the ideal gas law that mix \( V, n, \) and \( R \) in different ways.

The version we will be using in this lecture is: \( P = \frac{n}{m} k_B T \). We will use this expression to express the density \( \rho \) (mass of a molecule/volume of a molecule) as a function of \( P \).

\[ PV = N k_B T \]

\( N: \# \text{ molecules}, \quad k_B = 1.38 \times 10^{-23} \text{J/K} \)

From Collisions to Pressure and the Ideal-Gas Law

We can use the fact that the pressure in a gas is due to the collisions of particles with the walls to make some qualitative predictions. Figure 12.8 presents a few such predictions.

**FIGURE 12.8 Relating gas pressure to other variables.**

- Increasing the temperature of the gas means the particles move at higher speeds. They hit the walls more often and with more force, so there is more pressure.
- Decreasing the volume of the container means more frequent collisions with the walls of the container, and thus more pressure.
- Increasing the number of particles in the container means more frequent collisions with the walls of the container, and thus more pressure.

Based on the reasoning in Figure 12.8, we expect the following proportionality:

- Pressure should be proportional to the temperature of the gas: \( p \propto T \).
- Pressure should be inversely proportional to the volume of the container: \( p \propto 1/V \).
- Pressure should be proportional to the number of gas particles: \( p \propto N \).

In fact, careful experiments back up each of these predictions, leading to a single equation that expresses these proportionalties:

\[ p = C \frac{NT}{V} \]

The proportionality constant \( C \) turns out to be none other than Boltzmann’s constant \( k_B \), which allows us to write

\[ PV = N k_B T \quad (12.14) \]

Ideal gas law, version 1

Equation 12.14 is known as the ideal gas law.
Equation 12.14 is written in terms of the number $N$ of particles in the gas, whereas the ideal-gas law is stated in chemistry in terms of the number $n$ of moles. But the change is easy to make. The number of particles is $N = nN_A$, so we can rewrite Equation 12.14 as

$$pV = nN_A k_B T = nRT$$  \hspace{1cm} (12.15)

Ideal-gas law, version 2

In this version of the equation, the proportionality constant—known as the gas constant—is

$$R = N_A k_B = 8.31 \text{ J/mol} \cdot \text{K}$$

The units may seem unusual, but the product of Pa and m$^3$, the units of $pV$, is equivalent to J.

Let's review the meanings and the units of the various quantities in the ideal-gas law:

- Number of moles in the sample or container of gas
- Absolute pressure (Pa)
- Gas constant, 8.31 J/mol $\cdot$ K
- Volume of the sample or container of gas (m$^3$)
- Temperature in kelvin (K)

**EXAMPLE 12.5** Finding the volume of a mole

What volume is occupied by 1 mole of an ideal gas at a pressure of 1.00 atm and a temperature of 0$^\circ$C?

**PREPARE** The first step in ideal-gas law calculations is to convert all quantities to SI units:

$$p = 1.00 \text{ atm} = 101.3 \times 10^3 \text{ Pa}$$
$$T = 0 + 273 = 273 \text{ K}$$

**SOLVE** We use the ideal-gas law equation to compute

$$V = \frac{nRT}{p} = \frac{(1.00 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(273 \text{ K})}{101.3 \times 10^3 \text{ Pa}} = 0.0224 \text{ m}^3$$

We recall from earlier in the chapter that 1.00 m$^3$ = 1000 L, so we can write

$$V = 22.4 \text{ L}$$

**ASSESS** At this temperature and pressure, we find that the volume of 1 mole of a gas is 22.4 L, a result you might recall from chemistry. When we do calculations using gases, it will be useful to keep this volume in mind to see if our answers make physical sense.

**Learning objectives:** After this lecture, you will be able to:

1. Explain why the pressure of a liquid varies linearly with depth.

2. Explain why the pressure in a gas varies exponentially with height.

3. Use the exponential pressure expression to solve problems involving gases.

4. Explain what causes fluids to flow.

5. Explain why the volume flow rate is constant for incompressible fluids and use the expression to solve for the flow characteristic in a pipe of varying dimensions.

6. Qualitatively explain the formation of aneurysms and its connection to fluid dynamics
Activity 5 (Class 8a): Capillary Rise

Let’s figure out why the column of water rises to a height $h$, and find an expression for $h$ in terms of $R$, $g$, $\rho$, and $\gamma$:

1. Explain why the pressure $P_{in}$, the pressure at point A, and the pressure at point B, are all equal.

2. What is $P_{in} - P_{out}$?
   Assume a hemispheric meniscus of radius $R$ & use the result for $\Delta P$ across a spherical interface/bubble (hint: it involves $\gamma$ and $R$)

   $P_{in} - P_{out} = \frac{2\gamma}{R}$

3. How does $P_{out}$ compare with: (i) the pressure $P_{in}$, (ii) pressure at point A, and (iii) pressure at point B? Is the pressure at $P_{out}$ higher, same, or lower than the pressure at these three other points?

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4. We are trying to find the height $h$ but the expression we found in 2 doesn’t yet depend on $h$.
   Using the hydrostatic equation $p_1 = p_0 + \rho gh$, find an expression for $P_{in} - P_{out}$ that includes the height $h$. hint: $P_A = P_{in}$.

   $P_{in} - P_{out} = P_A - P_{out} = \rho gh$

5. When you equate the results you found in (2) and (4) for $P_{in} - P_{out}$, you can now derive an expression for the capillary rise height $h$ of the fluid in the tube of radius $R$.

   $\frac{2\gamma}{R} = \rho gh$
   $h = \frac{2\gamma}{\rho g R}$

**Bonus:** Using the result in 5, find the capillary rise of water in a tube with radius: $R$=0.6 mm and $R$=0.15 mm. $\gamma_{H_2O} = 0.0728$ N/m and $\rho_{H_2O} = 1000$ kg/m$^3$. Do the tubes on the right contain water?

$h = 2 \cdot 0.0728/\rho_{H_2O} \cdot g \cdot R$

$R = 0.0006 m, h = 0.3 m$

$R = 0.0015 m, h = 0.025 m$

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**Activity 1: Pressure vs height/depth**

**Hydrostatic Pressure (Fluids)**

- **Air**
  - $P_0 = 1 \text{ atm} = 101.5 \text{ kPa}$
- **Water**
  - $\rho_{\text{water}} = 1000 \text{ kg/m}^3$

$$P_1 = P_0 + \rho_{\text{water}} g h$$

1. Sketch the pressure as a function of $y$, starting at the ground where the pressure is $P_0 = 1\text{ atm}$.

$$P_1 = P_0 e^{-\frac{y}{H}}$$

- $H = \frac{k_B T}{m g} \approx 8 \text{ km}$
- $m = 29 \text{ amu}$ on Earth

**Barometric Pressure (Air)**

- Scale height
- $H$ is distance to get to 37% of $P_0$

2. Sketch the pressure as a function of $h$, starting at the surface where the pressure is $P_0 = 1\text{ atm}$.

4. One of the plots above increases to the right, while the other decreases, why is that? One plot is straight and the other is curved, what qualitative difference in the fluids might lead to this difference?

3. **Sketch** the fluid density (as a function of $h$ and $y$, respectively) for each of the two situations above, starting at the surface.

   **Hint:** use the ideal gas law, assuming constant temperature

$$\rho = \frac{m}{k_B T} P$$
L8b: Am I getting it? #1

1. The picture shows Mount McKinley in Alaska with an elevation of \( h = 6200 \) m and Wonder Lake in the foreground with a depth of 85 m. Which expression corresponds to the pressure difference \( \Delta P \) experienced when moving from the bottom of Wonder Lake to the top of Mount McKinley? In other words, what is \( \Delta P = P_{\text{Bottom of Wonder Lake}} - P_{\text{Top of McKinley}}? \) The scale height of the atmosphere is \( H = 8 \) km, the height of Mount McKinley is \( \bar{h} = 6200 \) m, and depth of Wonder lake is \( D = 85 \) m.

A) \( \rho_{\text{water}}GD - 1 \text{atm} \cdot e^{-\frac{h}{H}} \)
B) \( 1 \text{atm} \cdot e^{-\frac{h}{H}} - \rho_{\text{water}}GD \)
C) \( \rho_{\text{water}}GD - \rho_{\text{air}}gh \)
D) \( \rho_{\text{air}}gh - \rho_{\text{water}}GD \)
E) \( 1 \text{atm} + \rho_{\text{water}}GD - 1 \text{atm} \cdot e^{-\frac{h}{H}} \)
F) \( \rho_{\text{water}}GD - 1 \text{atm} \cdot e^{-\frac{h}{H}} - 1 \text{atm} \)
Activity 2: Gas flow

So far we have considered only static fluids (i.e. fluids at rest). We will now study fluid dynamics.

1. Can you think of what may cause a fluid to flow? Explain. (Hint: What do you think causes wind?)
   1) Body force (gravity)
   2) Pressure force (change in pressure)
   3) Shear force (spread honey)

2. Why does air flow in and out of our lungs? Use the ideal gas law to make your argument.

   \[ PV = N k_B T \]

The heart: an amazing pressure pump! Pressure drives blood flow through vessels.
Activity 3: Volume flow rate

The volume flow rate \( Q \) (measure in \( m^3/sec \)) for incompressible fluids is constant anywhere along a tube, as shown on the right.

\[
Q = \frac{dV}{dt} = A \cdot \nu
\]

1. Please explain why the volume flow rate HAS TO BE constant, no matter the cross-sectional area \( A \) of a pipe. What would it mean if the flow rate were not constant along the pipe?

2. If the tube is 2cm in length and the cross-sectional area goes from \( A_1 = 4 \text{cm}^2 \) down to \( A_2 = 1 \text{cm}^2 \), what is the speed \( v_2 \) compared with the incoming speed \( v_1 \)?

\[
\frac{Q_1}{Q_2} = \frac{A_1 \cdot v_1}{A_2 \cdot v_2} = \frac{4 \text{cm}^2}{1 \text{cm}^2} = 4
\]

3. The volume flow rate of blood leaving the heart to circulate throughout the body is about \( 5 \text{L/min} (8 \times 10^{-5} \text{ m}^3/\text{s}) \) for a person at rest. All this blood eventually must pass through the smallest blood vessels, the capillaries. Microscope measurements show that a typical capillary is \( 3 \mu \text{m in radius} \), and the blood flows through it at an average speed of \( 1 \text{mm/sec} \). Estimate the average number of capillaries in the body.

\[
\frac{Q_{heart}}{Q_{capillaries}} = N \cdot \frac{A_{capillaries} \cdot v_{in \ capillary}}{8 \times 10^{-5} \text{m}^3/\text{s}} = \frac{N \cdot \pi \cdot r^2 \cdot 10^{-3} \text{m/s}}{3 \times 10^{-6} \text{m}}
\]

\[
N = \frac{3 \times 10^9 \text{ capillaries}}{N}
\]

**Bonus!** An average capillary is 1 mm long. How far (what distance) would all your capillaries arranged along a straight line run?

2000 miles
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One-Minute Paper

Your name: ___________________________  TF: ___________________________

Names of your group members: ___________________________

_____________________

_____________________

- Please tell us any questions that came up for you today **during** lecture. Write “nothing” if no questions(s) came up for you during class.

- What single topic left you **most confused** after today’s class?

- Any other comments or reflections on today’s class?