Lecture 8a: Surface Tension

- Pre-reading:

(a) Surface tension
Liquids tend to adopt shapes that minimize their surface area, for then the maximum number of molecules are in the bulk and hence surrounded by and interacting with neighbours. Droplets of liquids therefore tend to be spherical, because a sphere is the shape with the smallest surface-to-volume ratio. However, there may be other forces present that compete against the tendency to form this ideal shape, and in particular gravity may flatten spheres into puddles or oceans.

(b) Curved surfaces
The minimization of the surface area of a liquid may result in the formation of a curved surface. A bubble is a region in which vapour (and possibly air too) is trapped by a thin film; a cavity is a vapour-filled hole in a liquid. What are widely called 'bubbles' in liquids are therefore strictly cavities. True bubbles have two surfaces (one on each side of the film); cavities have only one. The treatments of both are similar, but a factor of $2$ is required for bubbles to take into account the doubled surface area. A droplet is a small volume of liquid at equilibrium surrounded by its vapour (and possibly also air).

The pressure on the concave side of an interface, $p_{\text{in}}$, is always greater than the pressure on the convex side, $p_{\text{out}}$. This relation is expressed by the Laplace equation,

$$p_{\text{in}} = p_{\text{out}} + \frac{2\gamma}{r} \quad (18.38)$$

The Laplace equation shows that the difference in pressure decreases to zero as the radius of curvature becomes infinite (when the surface is flat, Fig. 18.21). Small cavities have small radii of curvature, so the pressure difference across their surface is quite large. For instance, a 'bubble' (actually, a cavity) of radius 0.10 mm in champagne implies a pressure difference of 1.5 kPa, which is enough to sustain a column of water of height 15 cm.

(c) Capillary action
The tendency of liquids to rise up capillary tubes (tubes of narrow bore), which is called capillary action, is a consequence of surface tension. Consider what happens when a glass capillary tube is first immersed in water or any liquid that has a tendency to adhere to the walls. The energy is lowest when a thin film covers as much of the glass as possible. As this film creeps up the inside wall it has the effect of curving the surface of the liquid inside the tube. This curvature implies that the pressure just beneath the curving meniscus is less than the atmospheric pressure by approximately $2\gamma/r$, where $r$ is the radius of the tube and we assume a hemispherical surface. The pressure immediately under the flat surface outside the tube is $p$, the atmospheric pressure; but inside the tube under the curved surface it is only $p - 2\gamma/r$. The excess external pressure presses the liquid up the tube until hydrostatic equilibrium (equal pressures at equal depths) has been reached (Fig. 18.22).
To calculate the height to which the liquid rises, we note that the pressure exerted by a column of liquid of mass density $\rho$ and height $h$ is

$$p = \rho gh$$

(18.39)

This hydrostatic pressure matches the pressure difference $2\gamma/r$ at equilibrium. Therefore, the height of the column at equilibrium is obtained by equating $2\gamma/r$ and $\rho gh$, which gives

$$h = \frac{2\gamma}{\rho gr}$$

(18.40)

This simple expression provides a reasonably accurate way of measuring the surface tension of liquids. Surface tension decreases with increasing temperature (Fig. 18.23).

**Illustration 18.2 Calculating the surface tension of a liquid from its capillary rise**

If water at 25°C rises through 7.36 cm in a capillary of radius 0.20 mm, its surface tension at that temperature is

$$\gamma = \frac{1}{2} \rho gh r$$

$$= \frac{1}{2} \times (997.1 \text{ kg m}^{-3}) \times (9.81 \text{ m s}^{-2}) \times (7.36 \times 10^{-2} \text{ m}) \times (2.0 \times 10^{-4} \text{ m})$$

$$= 72 \text{ mN m}^{-1}$$

where we have used $1 \text{ kg m s}^{-2} = 1 \text{ N}$.

When the adhesive forces between the liquid and the material of the capillary wall are weaker than the cohesive forces within the liquid (as for mercury in glass), the liquid in the tube retracts from the walls. This retraction curves the surface with the concave, high pressure side downwards. To equalize the pressure at the same depth throughout the liquid the surface must fall to compensate for the heightened pressure arising from its curvature. This compensation results in a capillary depression.
Cohesion vs Adhesion

Water in a small glass tube will form a meniscus as shown in the figure above (right). This is because the water molecules are more attracted to each other (cohesion) than they are to the glass container (adhesion). By forming the shape of a meniscus, the water molecules can increase the amount of water touching the glass.

KEY: (1) Cohesion is related to surface tension
     Liquid-air interface
(2) Adhesion is at solid-liquid interface

- *Hydrophobic* means cohesion of water is stronger than adhesion to surface
- *Hydrophilic* means adhesion to surface is stronger than cohesion of water

Cohesion in liquid mercury is very strong, while its adhesion to glass is weak. If mercury is poured into a glass-graduated cylinder, which drawing best describes the meniscus? Explain.

A)  B)  C)  D)

The answer is ...C. The mercury atoms are more attracted to each other, compared to their attraction to the glass, so by forming into the shape in C, they decrease the amount of mercury that must touch the glass.
Learning objectives: After this lecture, you will be able to:

1. Explain the concept of surface tension
2. Explain that fluid surfaces exert forces and, as a consequence, tend to minimize their area.
3. Describe the simple surface tension model used to explain disparate phenomena
4. Qualitatively and quantitatively describe the pressure inside curved surfaces
5. Explain capillary rise and wetting of solid surfaces
6. Explain the effect of surfactants on liquids and know some of its important applications
Physics of Surfaces and Thin Films

The surface or interface between a liquid and gas is always under tension: the surface acts as if there is an elastic membrane at the interface. Let’s take a look at some examples…

Activity 1: Modeling Surface Tension

In a liquid, each molecule feels an attractive force from each of its neighbors. If a molecule is in the bulk of the fluid, it will feel balanced attractive forces from all sides as shown in the figure at right.

1. The figure below shows a rectangular “drop” of liquid. For each of the highlighted molecules, what is the direction of the net force due to its interactions with all the neighboring molecules? (Choose the closest arrow out of the eight choices in the figure above.)

2. Explain why a drop of water will actually adopt a spherical shape (not a rectangle…)
Activity 2: Forces at a planar surface

1. In the video, the moveable wire between two soap films was initially in static equilibrium. The soap films exert tension forces on the wire.

(a) Draw a FBD of the wire showing the directions of the forces exerted by the soap films on the wire. (Note that the rectangular frame supplies the normal force on the wire.)

(b) What must be true about the tension forces from the soap if the wire is in equilibrium?

2. Does the force on wire depend on the area of the soap film? Explain how you know. (Hint: compare the area of the soap film on the left vs. the right)

3. When the film on the right is removed (by popping it), the wire accelerates to the left. A series of experiments on different wires finds that the initial acceleration on a wire of mass \( m \) and length \( L \) is given by:

\[
a = \frac{2\gamma L}{m}
\]

The constant \( \gamma \) is the surface tension of the soap/air interface.

(a) From this equation, find the net force on the wire, and determine the SI units of \( \gamma \). (leave N, Newtons, in your expression)

Bonus! If you could measure the force exerted by a single soap/air interface (e.g. the top surface of the soap film alone) on a wire of length \( L \), what would it be?
Activity 3: Surface Tension and Surface Energy

1. Another way to look at surface tension is to consider the energy due to interactions between molecules. Between every pair of neighboring molecules there is a favorable (negative) energy of interaction. For the liquid shown below, rank the four indicated molecules in order from lowest energy to highest energy. (Consider only the energy of molecular interactions.)

2. If you can rearrange the molecules in a liquid in any way, what arrangement would give the lowest energy? (Choose one and explain your choice)
   a) As many molecules as possible on the surface
   b) As few molecules as possible on the surface
   c) Half the molecules on the surface and half in the bulk

3. Give an argument in terms of energy for why raindrops are round (not cubes, or ellipsoids…)

4. We’re going to suspend a loop of thread on a soap film, as shown, and then puncture the film inside the loop. What will happen to the thread/loop? Explain…
   a) it will shrink
   b) it will expand
   c) it will fall

Bonus! We’re going to take a wire tetrahedron (shown at right) and stretch a soap film such that the film is attached to all the wire edges. What will be the shape of the film? (Hint: what does the soap film “want” to do?)
Pressure in a bubble

Consider gas bubble in a liquid. As a result of the surface tension squeezing the bubble (like a rubber balloon), the pressure inside the bubble is higher than the pressure outside of the bubble.

The difference in pressure across any spherical interface, of radius $R$, between two fluids is given by:

$$P_{\text{in}} - P_{\text{out}} = \frac{2\gamma}{R}$$

*note: this kind of bubble is often referred to as a “cavity” because “bubble” can also describe a soap film with air both inside and outside.

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L8a: Am I Getting It?

1. Two bubbles of different sizes are filled with air and connected by a tube. When we allow air to flow from one bubble to the other, what will happen? Explain.
   a) The big bubble will get bigger
   b) The two bubbles will end up becoming equal in size
   c) Nothing will change

2. Water in a cup is open to the air above. Four points are labeled in the diagram; point D is inside a small air bubble at the bottom of the cup. At which point is the pressure the highest? Explain.
Activity 4: Capillary rise

Let’s figure out why the column of water rises to a height $h$:

1. Explain why the pressure $P_{in}$, the pressure at point A, and the pressure at point B, are all equal.

2. What is $P_{in} - P_{out}$?
   Hint: assume a hemispheric meniscus & use the result from the previous activity.

3. How does $P_{out}$ compare with: (i) the pressure $P_{in}$, (ii) pressure at point A, and (iii) pressure at point B? Is the pressure at $P_{out}$ higher, same, or lower than the pressure at these three other points?

4. We are trying to find the height $h$ but the expression we found in 2 doesn’t yet depend on $h$. Using the hydrostatic equation $p_1 = p_0 + \rho gh$, find an expression for $P_{in} - P_{out}$ that includes the height $h$. Hint: $P_{in} = P_A$.

5. When you equate the results you found in (2) and (4) for $P_{in} - P_{out}$, you can now derive an expression for the capillary rise height $h$ of the fluid in the tube of radius $R$.

Bonus: Using the result in 5, find the capillary rise of water in a tube with radius: $R=0.6$ mm and $R=0.15$ mm. $\gamma_{H_2O} = 0.0728$ N/m and $\rho_{H_2O} = 1000$ kg/m$^3$. Do the tubes on the right contain water?
Surfactants and Respiratory Distress Syndrome

Surfactant = Surface active agent – a substance that reduces the surface tension of the liquid (compared with the pure liquid).

Common Surfactants: soap, detergent, alcohol

Example: A bead of liquid rests on a solid surface, spreading out on the substrate over an area $A$. If a surfactant is introduced into the liquid, will the area of contact increase, decrease, or stay the same? Answer: increase.

Try it! You can try the above with a bead of water on a plate. Now add soap (soap is a surfactant)

Crucial medical application: Neonatal respiratory distress syndrome. Lungs in premature babies haven’t started to produce surfactant, so it is difficult to fully inflate the alveoli (air sacs inside lungs). Total surface area is 80 m$^2$, so without surfactant the surface energy is extremely high. Today, we can treat this condition with synthetic surfactants.

Respiratory distress syndrome was the cause of many infant deaths, including JFK’s son Patrick:

The Boston Globe

BABY SPED TO BOSTON

Has Trouble Breathing; Kennedy Stands by Here

DONALD S. GROFF

Patrick Joseph Kennedy was the first child ever to leave Boston General in order to 40 years of neonatal intensive care.

Preparation of the baby's condition and diagnosis of his birth weight, 3.2 kg, was made by Dr. Robert F. Kennedy, who delivered the baby at Massachusetts General Hospital.

Mrs. Kennedy, who is married to JFK, has been a frequent visitor to the hospital and has visited with her husband on numerous occasions.

The baby was delivered by Dr. Charles A. M. Brown, who performed the same operation on JFK's son, Patrick, last year.

“We are very happy that the baby is doing well,” said Mrs. Kennedy, who arrived in Boston last night with her husband.

JFK was on hand last night to visit with the baby and to meet the family.

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Alveolar type II cell

Alveolar type I cell

Lamellar body

Surfactant layer

Air space

Tubular myelin

Alveolar fluid

Alveolar macrophage

Biological example: Alveoli in lungs

www.patient.co.uk

Surfactants (like soap) are amphiphilic molecules that like to adsorb on interfaces. They act to lower the surface tension. Lack of lung surfactant -> more work needed to expand alveoli and breathe.
One-Minute Paper

Your name: _____________________________       TF: ______________________________

Names of your group members:  __________________________________
                                    __________________________________
                                    __________________________________

• Please tell us any questions that came up for you today during lecture. Write “nothing” if no questions(s) came up for you during class.

• What single topic left you most confused after today’s class?

• Any other comments or reflections on today’s class?