Physical Sciences 2: Assignments for Oct 30 - Nov 8 2018
Homework #8: Surface Tension, Gases, and Fluid Flow
Due Thursday, Nov. 8, at 9:00AM

This assignment must be turned in by 9:00AM on Thursday, November 8. Late homework will not be accepted. Please write your answers to these questions on a separate sheet of paper with your name and your section TF’s name written at the top. Turn in your homework to your section TA’s box.

You are encouraged to work with your classmates on these assignments, but please write the names of all your study group members on your homework.

After completing this homework, you should…

- Understand surface tension.
- Be able to explain how fluid surfaces exert forces.
- Be able to qualitatively and quantitatively describe the pressure inside curved surfaces.
- Be able to explain capillary rise and the wetting of solid surfaces.
- Be able to explain the effect of surfactants on liquids and some important applications.
- Understand why the pressure of a liquid varies with depth.
- Understand why the pressure in a gas varies exponentially with height.
- Be able to use the exponential pressure expression to solve problems involving gases.
- Know what causes fluids to flow.
- Understand why the volume flow rate is constant for incompressible fluids, and use the expression to solve for flow characteristics in a pipe of varying dimension.
- Be able to qualitatively explain the formation of aneurysms and its connection to fluid dynamics.
Surface Tension

- Liquids adopt shapes that minimize surface area so that the maximum number of molecules are located in the bulk (i.e., not on the surface) of the liquid.
- Surface tension is caused by cohesive forces between molecules in the liquid.
- The magnitude of the tension force along some length \( L \) is
\[
F_T = \gamma L
\]
where \( \gamma \) is the surface tension of the interface.

Laplace Equation

- The minimization of surface area could cause a curved surface to form.
- Laplace equation gives the relationship between the pressure change across the interface, surface tension, and the radius of the curved surface:
\[
\frac{P_{\text{in}} - P_{\text{out}}}{\gamma} = \frac{2}{R}
\]

- Does a bigger or smaller bubble have a larger internal pressure?
\[
\begin{align*}
P_{\text{in}} - P_{\text{out}} &= \frac{2\gamma}{R} \\
\frac{P_{\text{in}}}{\text{concentric}} &= \frac{P_{\text{out}}}{\text{convex}} \\
\frac{P_{\text{in}}}{\text{inner}} &= P_{\text{out}} + \frac{2\gamma}{R} \\
\text{since } r < R &\Rightarrow P_{\text{inner}} > P_{\text{outer}} \Rightarrow \text{Pressure greater inside smaller bubble}
\end{align*}
\]
Capillary Action
- Capillary action is the tendency of liquids to rise up capillary tubes.
- How high does the liquid rise?
  - Adhesion between the liquid and glass causes a curved surface (meniscus), pressure difference given by Laplace's equation:
    \[ P_{\text{in}} - P_{\text{out}} = \frac{2\gamma}{R} \]
    \[ P_{\text{in}} - P_{\text{out}} + \frac{2\gamma}{R} \text{ is the radius of capillary tube} \]
  - From Pascal's principle, pressure is the same along a horizontal line, so \( P_a = P_{\text{out}} \) and is also equal to
    \[ P_a = P_{\text{out}} + \rho g h \]
  - Set these expressions equal to each other and solve for \( h \):
    \[ P_{\text{in}} = P_a \]
    \[ P_{\text{in}} = P_{\text{out}} + \frac{2\gamma}{R} \]
    \[ h = \frac{2\gamma}{\rho g R} \]

Pressure vs. Height
- How pressure varies with respect to height is different for liquids and gases.
  - Liquids are incompressible, so \( S_g \) is constant, \( S_p \) can change for gases.
    \[ P(h) = P(o) + S_p h \]
    \[ \frac{dP}{dh} = S_p \]
  - Gases (ideal gas law):
    \[ P = \frac{m}{n} k_B T \]
    \[ \frac{dP}{dh} = -S_p = -\left(\frac{m g}{k_B T}\right) \]
    \[ P(h) = P(o) e^{-\frac{m g h}{k_B T}} = P(o) e^{-S_p h} \]

Fluid Flow
- For an incompressible fluid, the volume flow rate \( Q \) must be constant anywhere in a tube.
- From conservation of mass, amount of fluid entering tube must equal amount leaving tube in cases of steady flow.
  \[ A_1 v_1 = A_2 v_2 \]
0. Reflections on Last Assignment (1 pt)

Pick one question from Homework 4 that you found particularly difficult and

a) describe any mistakes or misunderstandings you made

b) describe the best strategies to ensure you learn from your mistakes and won’t have the same misunderstanding again

1. Surface energy (2 pts).

Let’s use the concept of surface tension as surface energy per unit area to see if we can estimate, at least to the correct order of magnitude, the surface tension of water.

a) Water has a molar mass of 18 g/mol and a density of 1000 kg/m³ (or 1 g/cm³). Based on this data, estimate the number of water molecules per unit surface area of water.

b) The coordination number of water (i.e., the average number of “neighbors” each water molecule has) in the liquid state is 4. Neighboring water molecules attract each other via hydrogen bonds, each of which has a binding energy of roughly $10^{-20}$ J (although this number depends relatively strongly on temperature). Use this information to estimate the surface tension of water. How does your estimate compare to the observed figure ($\gamma_{\text{water}} = 0.072$ N/m)?

2. Xylem and phloem (1 pts)

Water and nutrients are delivered from the roots to the leaves in trees via tube-like tissues known as xylem and phloem. If these tissues have radii around 100 μm, calculate the capillary rise for water in the xylem of a tree. Does this answer make sense? If not, propose another hypothesis for the transport of sap in trees. *Hint: what are the radii of the pores, or stomata, in the leaves?*

3. Water strider (2 pts)

The end of a water strider’s foot can be treated as roughly spherical with a radius of $r = 20$ μm. Let’s look at the forces on a sphere which is partially submerged in water, as in the diagram at right. Use $\gamma_{\text{air-water}} = 0.07$ N/m.

a) If the foot is submerged such that the radius to the level of water makes an angle $\theta$ with the vertical, as shown, what is the net force on the foot due to surface tension, in terms of $\gamma$, $r$, and $\theta$? For what value of $\theta$ is this a maximum?
b) Calculating the buoyant force on the sphere requires difficult calculus to determine the volume of just the underwater portion of the sphere. However, we can at least estimate its order of magnitude: if exactly half of the sphere is underwater, what is the buoyant force on the sphere? How does this compare in magnitude to the force due to surface tension in that position? Use $\rho_{\text{water}} = 1000 \text{ kg/m}^3$.

c) A water strider has a mass of about 3 milligrams and has six legs. What will be the angle $\theta$ when the strider is resting on the water’s surface?

4. Blood flow (2 pts)

When you are resting, your heart typically pumps about 5 L of blood through your body every minute.

a) Your aorta has a radius of about 0.9 cm. What is the speed of blood flow in your aorta?

b) Your lungs are designed with tremendous redundancy; they have far more capillaries than are actually used at any one time. (The remaining capillaries are collapsed and do not support blood flow.) Each of your 300 million alveoli is surrounded by an estimated 1000 tiny capillaries. Each capillary has a radius of approximately $4 \times 10^{-4}$ cm, and blood flows in a capillary at a speed of about $5 \times 10^{-4}$ m/s. Estimate the fraction of capillaries in your lung that are actually being used.

5. Breathe easily (2 pts)

Your lungs contain about 300 million tiny alveoli, which are spherical sacs that inflate with air. You can think of each alveolus as a tiny bubble of air surrounded by fluid. The fluid is ordinary mucous tissue fluid that has a surfactant added to lower its surface tension; the resulting surface tension is about $\gamma = 3 \text{ mN/m}$.

a) Newborn babies must create a pressure difference between the inside and outside of the alveoli of about 30 torr to expand their alveoli for the first time. Estimate the radius of an alveolus before it is inflated for the first time in a newborn baby.

b) Some newborns do not have the surfactant needed in their alveoli, so the surface tension of the mucous tissue fluid is increased by a factor of about 15. This condition, known as respiratory distress syndrome, means that tremendous pressure is required in order for the newborn to take her first breath. What pressure would be required to inflate the alveoli in the absence of surfactant?