MATHEMATICS 23a, FALL 2018
Linear Algebra and Real Analysis I

Last revised: October 24, 2018
Course Website: `https://canvas.harvard.edu/courses/45219`

Introductory meeting: Tuesday, September 4, at 8:15 AM, in Science Center Hall A. If there are lots of shoppers, you might need to arrive early to get a seat.

Instructor: Paul Bamberg (to be addressed as "Paul," please)
Paul graduated from Harvard in 1963 with a degree in physics and received his doctorate in theoretical physics at Oxford in 1967. He taught in the Harvard physics department from 1967 to 1995 and joined the math department in 2001. From 1982 to 2000 he was one of the principals of the speech recognition company Dragon Systems. If you count Extension School and Summer School, he has probably taught more courses, in mathematics, physics, and computer science, than anyone else in the history of Harvard. He was the first recipient of the White Prize for excellence in teaching introductory physics.
Email: `bamberg@tiac.net`
Office: Science Center 322, (617-49)5-9560

Office Hours:
Tuesday, 2:00-3:00 in Science Center 322.

Sections:

Code	Day	Start	End	Workshop	Location	Specialty
*D*003	*Thursday*	3:00	3:55	3:55-4:50	Northwest B-106	Data, Computing
*R*003	*Thursday*	3:00	3:55	3:55-4:50	Northwest B-107	Physics
*R*004	*Thursday*	4:50	5:45	3:55-4:50	Northwest B-107	Physics
*D*005	*Friday*	12:00	12:55	12:55-1:50	Northwest B-106	Data, Computing
*R*005	*Friday*	12:00	12:55	12:55-1:50	Northwest B-110	Math
*Z*005	*Friday*	12:00	12:55	12:55-1:50	Vanserg 214	Economics
*D*006	*Friday*	1:50	2:45	12:55-1:50	Northwest B-106	Data, Computing
*R*006	*Friday*	1:50	2:45	12:55-1:50	Northwest B-107	Economics
*Z*006	*Friday*	1:50	2:45	12:55-1:50	Northwest B-110	Math
*D*004∗	*Thursday*	5:00	5:55	3:55-4:50	Northwest B-110	Economics

The codes were assigned by the Registrar's office. They cannot be changed to something more meaningful. You will need to use them when you indicate section preferences in my.harvard.

How to get assigned to a section:

- Put Math 23a (Thursday or Friday, not both) in your Crimson Cart. If you have a choice, go for Friday. There is more space on Friday, and there is no more classroom space on Thursday for additional sections.
- Get your advisor to "sign" and move Math 23a into the enrolled courses section.
- I will approve your petition as quickly as I can.
- Go back into my.harvard and indicate section preferences (1 or 2 on Thursday, 2 or 3 on Friday)
- I will assign you to a section as quickly as I can.
- If you cannot get this done this week, just go to Northwest basement and find a room that has an empty seat.
- Section D004 will be offered only if the time conflict with Economics 10 creates an impossible situation. Last year folks could take Ec 10 Friday 12-1 and Math 23a Friday 1-3.

Head Teaching Assistant: Joe Palin (to be addressed as "Joe," please)
Email: jgpalin@gmail.com

Graduate Assistants:

- Weifeng Sun sun_weifeng@g.harvard.edu
- Kenneth Wang kwang02@g.harvard.edu
- Zoe Hitzig zhitzig@g.harvard.edu

Head Course Assistant: Becky Jarvis jarvis@college.harvard.edu

Course Assistants:
(all former students and/or instructors in Math 23a)

- Gerardo Ascensio gascencio@college.harvard.edu
- Tushar Dwivedi tushar_dwivedi@college.harvard.edu
- Maryam Hiradfar maryamhiradfar@college.harvard.edu
- Annika McDermott-Hinman mcdermotthinman@college.harvard.edu
- Claire Millett clairemillett@college.harvard.edu
- Angela Tang angelatang@college.harvard.edu
- David Xu davidxu@college.harvard.edu

Goals: Math 23a is the first half of a moderately rigorous course in linear algebra and multivariable calculus. It is designed for students who are serious about mathematics and interested in being able to prove the theorems that they use, but who are as much concerned about the application of mathematics in fields like physics and economics as about "pure mathematics" for its own sake. Trying to cover both theory and practice makes for a challenging course with a lot of material, but it is appropriate for the audience!

Prerequisites: This course is designed for the student who received a grade of 5 on the Math BC Advanced Placement examination or an equivalent score on the Harvard placement test. Probably the most important prerequisite is the attitude that mathematics is fun and exciting.

Our assumption is that the typical Math 23a student knows only high-school algebra and single-variable calculus and is currently better at formula-crunching than at doing proofs. We do not assume that Math 23 students have any prior experience in either of these areas beyond solving systems of linear equations in high school algebra.

We expect expertise in the mechanics of single-variable differential and integral calculus, but no more. You sholud have high scores on Parts I and II and on the integration section of Part III. The rest of Part III is almost irrelevant.

We will devote four weeks to single-variable real analysis. Real analysis is the study of real-valued functions and their properties, such as continuity and differentiability, as well as sequences, series, limits, and convergence. This means that if you had a calculus course that touched only lightly on topics like series, limits, and continuity, or if you are an international student whose calculus course omitted these topics, you will be OK.

Every year there are rumors like

- "You need to have studied multvariable calculus before taking Math 23."
- "You need previous experience with proofs in order to take Math 23."
- "You need to have taken a course in linear algebra before Math 23."

Such rumors are wrong! This is a fast-paced and challenging course, but it presupposes only solid precalculus and single-variable calculus.

A rumor that would probably be well-founded is
"Most students in Math 23 have scored 27 or more (out of 30) on part I of the placement test (the precalculus part)."

Indeed, most students in the course are remarkably good (perhaps better than the instructor) at doing mathematics without making careless errors.

Classes

Videos have been made of all the lectures, and these have been posted on the Web site. The course materials for each week list the relevant videos.

There is also an interactive class on each week's material.

Classes meet on Thursday 3-5:45 or Friday noon-2:45 PM, at the end of the week in which you are expected to watch the lectures. Members of the class will present five seminar topics, which go over key definitions and proofs, during one hour. The other hour will be devoted to problem solving in small groups. The two activities can be done in either order and will use only two hours. Lecture videos are crucial background for class!

When you register, choose either the Thursday meeting pattern or the Friday meeting pattern. There is more room available on Friday; choose it if possible.

Sections will meet the first week, on September 6 and 7. Please try to get your advising hold lifted ASAP so that you can join the course officially and be assigned to a section. If this is not possible, just show up in Northwest B-100 at the right time.

Before coming to the first class, print the Week 1 materials, watch the first two lectures, and volunteer on the Web site to present a seminar topic. The term is short, and we cannot afford to waste the first week!

For each lecture, there is also a scan of Paul's lecture notes on the Web site. Consensus is that you do will do best to take your own notes and use these scanned notes only as backup.

Most students report that it is efficient to play the easy parts of the lecture at 1.5 times normal speed, then to slow down and perhaps even to run twice though difficult proofs or intricate examples.

Exams: There will be two quizzes and one final exam.

Quiz 1:	second weekend in October (linear algebra, weeks 1-4)
Quiz 2:	second weekend in November real analysis, weeks 5-8)
Final Exam:	December 17 (multivariable analysis and calculus, weeks 9-12)

If you are on campus during a quiz weekend, take the quiz on Saturday morning between 8 AM and 1 PM. Start before 10:30; end after 10:30. You will need about two hours. THe first quiz is in Jefferson 250.
The second quiz is in the Geological Lecture room (since the Harvard-MIT math tournament reserved every other large room on campus).

Quizzes can also be taken off-campus. You will need to find a informal proctor (family members or roommates are OK) who will sign a statement that you completed the quiz under closed-book conditions, with computers and cell phones turned off. You must complete the quiz at a single sitting during a specified 24-hour period, but there is otherwise no time limit. Two hours should be sufficient.

Quizzes will include questions that resemble the workshop problems done in class, and each quiz will include two or three randomly-chosen proofs from among the numbered proofs in the relevant modules. There may be other short proofs similar to ones that were done in lecture and problems that are similar to homework problems.

The final examination will focus on material from the last five weeks of the course. The Registrar's office has cooperated by making it late in exam period.

If you have an unexpected time conflict for one of the quizzes, contact Paul as soon as you know about it, and special arrangements can be made.

Textbooks:

Vector Calculus, Linear Algebra, and Differential Forms, Hubbard and Hubbard, fifth edition, Matrix Editions. We will cover only Chapters 1-3. If you plan to continue in Math 23b or 23c in the spring, be sure to get the current (fifth) edition. A used copy of the fourth edition, if you find one at an attractive price, will suffice for the fall term.

Ross, Elementary Analysis: The Theory of Calculus, 2nd Edition, 2013.

This will be the primary text for the module on single-variable real analysis. It is available electronically through the Harvard library system (use HOLLIS and search for the author and title). If you like to own bound volumes, used copies can be found on amazon.com for as little as $25, but be sure to get the correct edition!

Lawvere, Conceptual mathematics: a first introduction to categories, 2nd Edition, 2009.

We will be using only the first chapter. The book is no longer available for free download through the Harvard library system. See the Web site for a workaround.

Proofs:

Learning proofs can be fun, and we have put a lot of work into designing an enjoyable way to learn high level and challenging mathematics! Each week's course materials includes two proofs. Often these proofs appear in the textbook and will also be covered in lecture. They also may appear as quiz questions.

You can earn points towards your grade by presenting these proofs to teaching staff and to each other without the aid of your course notes. Here is how the system works:
When we first learn a proof in class, only members of the teaching staff are "qualified listeners." Anyone who presents a satisfactory proof to a qualified listener also becomes qualified and may listen to proofs by other students. This process of presenting proofs to qualified listeners occurs separately for every proof.

You are expected to present each proof before the date of the quiz on which it might appear; so each proof has a deadline date.

Each proof is worth 1 point. Here is the grading system:

- Presenting a proof to Paul, Joe, one of the course assistants, or a fellow student who has become a qualified listener: 0.95 points before the deadline, 0.8 points after the deadline. You may only present each proof once.

- Listening to a fellow student's proof: 0.1 point. Only one student can receive credit for listening to a proof.

- After points have been tallied at the end of the term, members of the course staff may assign the points that they have earned by listening to proofs outside of section to any students that they feel deserve a bit of extra credit.

Students who do the proofs early and listen to lots of other students' proofs can get more than 100%, but there is a cap of 30 points total.You can almost reach this cap by doing each proof before the deadline and listening twice to each proof.

Either you do a proof right and get full credit, or you give up and try again later. There is no partial credit. It is OK for the listener to give a couple of small hints.

You may consult the official list of proofs that has the statement of each theorem to be proved, but you may not use notes. That will also be the case when proofs appear on quizzes and on the final exam.

It is your responsibility to use the **proof logging software that will be posted on the course Web site** to keep a record of proofs that you present or listen to. You can also use the proof logging software to find listeners for your proofs. A link will be posted on the course web site in mid-September.

Paul will hold several "Speak Proof to Power" sessions on various Mondays and Wednesdays. Sign up for a 15-minute time slot and present one or two proofs at the blackboard in Science Center 322. If you hope to become a course assistant next year or will need a recommendation, it is a good idea to participate. Even at just 0.1 points per proof, Paul accumulates quite a few listening points, which can be used to fudge borderline grades for students who have participated.

Each quiz will include three questions (answer two) which are proofs chosen at random from the four weeks of relevant material. The final exam will have three proofs, all from material after the second quiz. Students generally do well on the proof questions.

Alternatively, you may opt out of the proof logging system. In that case you will need to do one more proof on each exam, and you will be expected, over the course of the term, to upload eight proofs to the course Web site.

Advice for the grade-conscious:

It is easy for a diligent student to get more than 100% for proof logging. However, if you do not present proofs before the deadline, you would probably have been better off just relying on homework and exams.

Useful software:

- R and RStudio
  This is designed for Extension students who register for graduate credit, but it is an option for everyone. Consider learning R if you

  - are interested in computer science and want practice in using software to do things that are more mathematical than can be dealt with in introductory CS courses.
  - plan to take Math 23c in the spring term.
  - are thinking of taking a statistics course, which is likely to use R.
  - are hoping to get an interesting summer job or summer internship that uses mathematics or deals with lots of data.
  - want to be able to work with large data files in research projects in any field (life sciences, economics and finance, government, etc.)

  R is free, open-source software. Instructions for download and installation are on the Web site. You will have the chance to use R at the first class, so install it right away, preferably on a laptop computer that you can bring to class.

  On the course Website are a set of R scripts, with accompanying YouTube videos, that explain how to do almost every topic in the course by using R. These scripts are optional for undergraduate-credit students, but they will enhance your understanding both of mathematics and of R.

- LaTeX

  This is the technology that is used to create all the course handouts. Once you learn how to use it, you can create professional-looking mathematics on your own computer.

  The editor that is built into the Canvas course Web site is based on LaTeX.

  One of the course requirements is to upload four proofs to the course Web site in a medium of your choice. One option is to use LaTeX. Alternatively, you can use the Canvas file editor (LaTeX based), or you can make a YouTube video.

  I learned LaTeX without a book or manual by just taking someone else's files, ripping out all the content, and inserting my own, and so can you. You will need to download freeware MiKTeX version 2.9 (see `http://www.miktex.org`), which includes an integrated editor named TeXworks.

  From `http://tug.org/mactex/` you can download a similar package for the Mac OS X.

  When in TeXworks, use the Typeset/pdfLaTeX menu item button to create a .pdf file. To learn how to create fractions, sums, vectors, etc., just find an example in the lecture outlines and copy what I did. All the LaTeX source for lecture outlines, assignments, and practice quizzes is on the Web site, so you can find working models for anything that you need to do.

  The course documents contain examples of diagrams created using TikZ, the built-in graphics editor. It is also easy to include .jpg or .png files in LaTeX. If you want to create diagrams, use Paint or try Inkscape at `http://www.inkscape.org`, an excellent freeware graphics program. Students have found numerous other solutions to the problem of creating graphics, so just experiment.

Use of R:
You can earn "R bonus points" in three ways:

- By being a member of a group that uploads solutions to workshop problems that require creation of R scripts. These will be available most, but not all, weeks. (about 10 points)

- By submitting R scripts that solve the optional R homework problems (again available most, but not all, weeks). (about 20 points)

- By solving, before Thanksgiving, a set of 3 "R review problems" that cover linear algebra calculations that would be very tedious to do with pencil and paper (up to 6 points)

- By doing a term project in R. (about 20 points)

If you do everything, you will benefit from the "graduate credit" grade calculation. We wlil add in your R bonus points to the numerator of your score. To the denominator, we will add in 95% of your bonus points or 50% of the possible bonus points, whichever is greater. Earning more than half the bonus points is certain to raise your percentage score a bit, and it can make a big difference if you have a bad day on a quiz or on the final exam.

An alternative is to do R just for the linear algebra part of the course (weeks 1-4).

- By being a member of a group that uploads solutions to workshop problems that require creation of R scripts. (up to 5 points)

- By submitting R scripts that solve the optional R homework problems (up to 8 points)

- By solving, before Thanksgiving, a set of 3 "R review problems" that cover linear algebra calculations that would be very tedious to do with pencil and paper (up to 6 points)

If you choose this alternative, we will add your R bonus points to the numerator of your score. To the denominator, we will add in 95% of your bonus points or 15, whichever is greater.

Learning some R is strongly recommended if you are considering taking Math E-23c in the spring term or taking courses in data science.

Grades: Your course grade will be determined as follows:

- Problem sets, 50 points. Your worst score will be converted to a perfect score.
- Participation
  - volunteering to do seminar presentations, 10 points (1 per week, 10 of 13 weeks required)

    Posting a biography, arrival on time, good preparation and participation in class, 10 points.
- With proof logging
  - presenting and listening to proofs, 26 points (maximum 30 points).
  - two quizzes, 40 points each.
  - final exam, slightly more than 60 points.
- Without proof logging
  - uploading proofs to the Web site, 8 points.
  - two quizzes, 45 points each.
  - final exam, slightly more than 65 points.
- R bonus points (graduate), about 55 points in numerator, slightly less in denominator.
- R bonus points (undergraduate), about 19 points in numerator, slightly less in denominator.

The grading scheme is as follows:

Points	Grade
94.0%	A
88.0%	A-
80.0%	B+
75.0%	B
69.0%	B-
63.0%	C+
57.0%	C
51.0%	C-

If you are conscientious about the homework, proofs, and quizzes, you will end up with a grade between B plus and A, depending on your expertise in taking a fairly long and challenging 3-hour final exam, and you will know that you are thoroughly prepared for more advanced courses. For better or worse, you need to be fast as well as knowledgeable to get an A, but an A- is a reasonable goal even if you make occasional careless errors and are not a speed demon. There is no "curve" in this course! You cannot do worse because your classmates do better.

Last year was the first time that Math 23a used the current "flipped classroom" approach with lecture videos instead of live lectures. The most surprising result was a remarkable decline in the number of students who had disappointing results (below 80%) on exams.

Why? If you give lectures and make them available of videos, many students stop attending class and just watch the videos, and they may skip section meetings too. Under the current system, everyone turns up for class every week, and they are well prepared. So it is hard to fall seriously behind.

YouTube videos

- The Lecture Preview Videos were made by Kate Penner. They cover the so-called Executive Summaries in the weekly course materials, which go over all the topics, but without proofs or detailed examples.

  If you watch these videos (it takes about an hour per week) you will be very well prepared for the lecture videos, and even the most difficult material will make sense on a first hearing.

- The R script videos were made by Paul. They provide a line-by-line explanation of the R scripts that accompany each week's materials.

  If you are doing the "graduate" R option, these scripts are pretty much required viewing, although the scripts are so thoroughly commented that just working through them on your own is perhaps a viable alternative. For the undergraduate R option, the first four weeks suffice.

  If you are not looking for extra credit with R, you can ignore the R scripts completely.

Homework: Homework (typically 8 problems) is assigned weekly. The assignment is included in the same online document as the lecture notes and workshop problems.

Assignments are due on Tuesdays by 11:59 PM. Submit a .pdf file to the Assignments page on the course Web site. If you write your assignment with pencil and paper, you will need access to a scanner that can create a single .pdf file from all the pages.

Each week's assignment will include a couple of optional problems whose solutions require R scripts. These scripts should be uploaded electronically to the Web site each week. Please include your name as a comment in the script and also in the file name.

The course assistant who grades for your section should return your corrected homework to you electronically within a week after the due date. If you are not receiving graded homework on schedule, send email to Joe Palin and the problem will be dealt with.

Homework that is submitted after 11:59 PM on the Tuesday when it is due will not be graded. If it arrives before the final exam and looks fairly complete, you will get a grade of 50% for it.

Collaboration and Academic Integrity policy:

You are encouraged to discuss the course with other students and with the course staff, *but you must always write your homework solutions out yourself in your own words.* **You must write the names of those you've collaborated with at the top of your assignment.**

If you collaborate with classmates to solve problems that call for R scripts, create your own file after your study group has figured out how to do it.

Proofs that you submit to the course Web site must be done without consulting files that other students have posted!

If you have the opportunity to see a complete solution to an assigned problem, please refrain from doing so. If you cannot resist the temptation, you must cite the source, even if all that you do is check that your own answer is correct. In Fall 2017 two students were placed on probation for violating this rule!

You are forbidden to upload solutions to homework problems, whether your own or ones that are posted on the course Web site, to any publicly available location on the Internet.

Anything that you learn from lecture, from the textbook, or from working homework problems can be regarded as "common knowledge" for purposes of this course, and the source need not be cited. Anything learned in prerequisite courses falls into the same category. Do not assume that other courses use such an expansive definition of "common knowledge"!

Week-by-week Schedule:

Month	Date	Topic
Fortnight 1	September 4-16	Fields, vectors and matrices
Week 2	September 18-24	Dot and cross products; Euclidean geometry of $\mathbb{R}^n$
Week 3	September 25-Oct. 1	Row reduction, independence, basis
Week 4	October 2-8	Eigenvectors and eigenvalues
Week 5	October 9-15	Number systems and sequences
	October 13-14	QUIZ 1 on weeks 1-4
Week 6	October 16-22	Series, convergence tests, power series
Week 7	October 23-29	Limits and continuity of functions
Week 8	October 30-Nov. 5	Derivatives, inverse functions, Taylor series
Week 9	November 6-12	Topology, sequences in $\mathbb{R}^n$, linear differential equations
	November 10-11	QUIZ 2 on weeks 5-8
Week 10	November 13-19	Limits and continuity in $\mathbb{R}^n$; partial and directional derivatives
Fortnight 11	November 20-Dec 3	Differentiability, Newton's method, inverse functions, manifolds
	November 22	Thanksgiving
Week 12	December 4-10	Critical points, Lagrange multipliers
	December 17	FINAL EXAM on weeks 9-12