L7a: Am I getting it? #1

Recap from Lab: (Rolling without Slipping)

Rolling = Translation + Pure Rotation

\[ s = R \theta \]

\[ \omega = \frac{V_{\text{com}}}{R} \]

\[ v_{\text{com}} = \frac{ds}{dt} = \frac{d}{dt} (R \omega) \]

Kinetic Energy for Rolling Objects:

\[ K_{\text{tot}} = K_{\text{trans}} + K_{\text{rot}} = \frac{1}{2} M v^2 + \frac{1}{2} I \omega^2 \]

\[ \frac{1}{2} M v_{\text{com}}^2 + \frac{1}{2} \frac{I}{MR^2} v_{\text{cm}}^2 M = \frac{1}{2} M v_{\text{cm}}^2 \left( 1 + \frac{I}{MR^2} \right) \]

Three objects with same mass \( m \) are going down a distance \( L \) along the same ramp. There is no friction between the ice cube and the ramp. However, the cylinders roll without slipping. The moment of inertia for the solid and hollow cylinder are: \( I_{\text{solid}} = \frac{1}{2} m R^2 \) and \( I_{\text{hollow}} = m R^2 \), respectively.

\[ \Delta E_{\text{mech}} = 0 \Rightarrow \Delta K = -\Delta U = mg \Delta \theta \]

1. The ice cube will arrive at the bottom of the incline:
   a) Before the solid cylinder
   b) Same time as the solid cylinder
   c) After the solid cylinder
   d) I don’t know

2. The hollow cylinder will be arriving at the bottom of the incline:
   a) Before the solid cylinder
   b) Same time as the solid cylinder
   c) After the solid cylinder
   d) I don’t know

3. The first of the three objects arriving at the bottom of the incline is:
   a) Ice cube
   b) Solid cylinder
   c) Hollow cylinder
   d) I don’t know.

\[ \beta_{\text{cube}} = 0 \]

\[ \beta_{\text{solid}} = \frac{I_{\text{solid}}}{MR^2} = \frac{1}{2} \]

\[ \beta_{\text{hollow}} = \frac{I_{\text{hollow}}}{MR^2} = 1 \]

Continued on next page...
Activity 1: Generalized Hooke’s law for Elasticity

The stress-strain expression, relates the tension/compression force to the stretching or compression of materials. The stress is defined as \( \sigma = \frac{F}{A} \) and strain as \( \varepsilon = \frac{\Delta L}{L_0} \).

1. This relationship is none other than Hooke’s law for springs: \( F = -kx \), where ‘x’ and \( \Delta L \) are the same thing.

   a) Use the expression above to derive an expression for the spring constant \( k \).

   b) Use your value of \( k \) to derive an expression for the stored potential energy \( U(\Delta L) \). (Analog to \( U(x) = \frac{1}{2} kx^2 \).)

\[
U(\Delta L) = \frac{1}{2} k \text{eff} (\Delta L)^2 = \frac{1}{2} \left( \frac{YA}{L_0} \right) \Delta L
\]

2. If the stress-strain relationship is the same as Hooke’s law, where is the negative sign in \( \frac{F}{A} = Y \frac{\Delta L}{L_0} \)?

\[
\begin{align*}
\Delta x & \xrightarrow{\text{Force required to stretch rod by } \Delta L} \Delta L = k \text{eff} \Rightarrow k \text{eff} = \left( \frac{YA}{L_0} \right) \\
\text{stress} \rightarrow \text{strain} & = Y = \text{Young’s Modulus} \end{align*}
\]

3. Which part of this curve does the stress-strain formula apply?

Only in elastic region where \( F_{\text{app}} \propto \Delta L \) (i.e., linear in displacement)

**Bonus:** Why not just keep using Hooke’s law and apply it to elastic materials?
Physical Sciences 2: Lecture 7a

\[ \frac{F}{A} = \gamma \frac{\Delta L}{L} \rightarrow \sigma = \gamma \varepsilon \]

**Activity 2: Calculating stress-strain**

1. A force stretches a wire by 1mm.

a) A 2\textsuperscript{nd} wire of the same material has the same cross section $A$ and twice the length $L$. How far will the same force stretch it? Explain.

\[
\Delta L_1 = \left(\frac{FL}{AY}\right) L_1 \\
\Delta L_2 = \Delta L_1 \left(\frac{L_2}{L_1}\right) \\
= 2 \Delta L_1 \\
= 2 \text{mm}
\]

b) A 3\textsuperscript{rd} wire of the same material has the same length $L$ and twice the diameter of the first. How far will the same force stretch it? Explain.

\[
\Delta L_{(j)} = \frac{FL}{Y \left(\frac{L}{A_{(j)}}\right)} \\
\Delta L_2 = \frac{1}{4} \Delta L_1 = 0.25 \text{mm}
\]
Activity 3: Elevator cable stretch

The steel cables that hold elevators stretch only a very small fraction of their length, but in a tall building this fractional change can add up to a noticeable stretch. The 2000 kg car of a high speed elevator in a tall building is supported by six $d = 0.01$ m diameter cables. Young’s modulus for the cables is $Y = 10 \times 10^{10}$ N/m$^2$. When the elevator is in the bottom floor, the cable rise 100 m up the shaft to the top floor ($L = 100$ m).

$$A = \frac{\pi d^2}{4} = \frac{\pi}{4} \times (0.01)^2 \text{ m}^2$$

With 20 people on board having a combined mass of 2000 kg, the elevator accelerates upward at 2 m/s$^2$ until it reaches cruising speed.

a) How much do the cables stretch due to the weight of the car alone? (Assume the elevator is at rest). How much additional stretch occurs when passengers are in the car?

b) What is the total stretch of the cables when the car is accelerating upward at 2 m/s$^2$? Is your answer larger, same, or smaller than in (b). Explain your answer.

**Bonus:** What is the stretch in the cables if the elevator is accelerating downward at -2 m/s$^2$? Compare your answer with (b) and (c) and discuss.

$$a_y = -2 \text{ m/s}^2$$

$$\Delta L = \frac{m(g+a_y)L}{YA} < \Delta L(a) < \Delta L(b)$$