物理科学2: 作业指南（10月23日至11月1日，2018年）

作业#7: 弹性与流体静力学

due Thursday, Nov 1st, at 9:00AM

这份作业必须在周四，11月1日9:00AM之前提交。迟交的作业将不被接受。请将你的答案写在另一张纸上，上面写着你的名字和你的TF的名字。将你的作业交给指定的TA的盒子。

你可以和你的同学们一起完成这些作业，但请写上所有你研究小组成员的名字。

完成这份作业后，你应该:
- 能够描述什么是弹性以及它如何与胡克定律相关
- 能够解释为什么需要杨公式，以及为什么胡克定律不能总是使用
- 理解应力-应变公式，并能解释所有术语的含义
- 能够解释应力-应变图
- 能够解释流体的静力性质：密度，可压缩性，和压力
- 理解帕斯卡原理，并能够应用它
- 能够解释压力如何以及为什么在流体中变化，以及它的后果（浮力）
- 知道阿基米德原理，并能够应用它来解决涉及浮力问题
Here are summaries of this module’s important concepts to help you complete this homework:

Module 7: Elasticity and Fluid Statics
Compiled by Kristina Callaghan

**Elasticity**
- any material can stretch due to its spring-like molecular bonds
  - elastic limit: maximum $\Delta L$ for which the material will return to initial length $L$; past this point, the object will permanently deform and/or break
  - the relationship between the force applied and the change in distance is
    \[ \frac{F}{A} = Y \cdot \frac{\Delta L}{L} \]
  - this has the same form as Hooke’s law:
    \[ F = \frac{Y \cdot A}{L} \cdot \Delta L \]
  - positive RHS because $F$ and $\Delta L$ in same direction
  - stress is the quantity $F/A$
  - strain is the quantity $\Delta L/L$

**Fluid Statics**
- density $\rho$ is the ratio of mass $m$ and volume $V$, and is an intrinsic property of the material
  \[ \rho = \frac{m}{V} \]
  - Density of water, $\rho_{\text{water}} = 1 \text{ g/cm}^3 = 1000 \text{ kg/m}^3$
- a fluid is incompressible, meaning its molecules are already as close to each other as possible without touching
- pressure $P$ is the amount of force exerted over some cross-sectional area $A$
  \[ P = \frac{F}{A} \]
- Units: 1 Pascal (Pa) = 1 N/m$^2$; 1 atm = 101.3 kPa; 1 mm Hg = 1 atm = 760 mm Hg

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Fluid Statics

Pascal’s Principle: pressure is transmitted undiminished in an enclosed static fluid

Example: consider an object at depth $h$ above the liquid is some pressure $P_i$

![Diagram of Pascal's Principle]

The pressure at depth $h$, denoted as $P_h$, is equal to the pressure $P_0$ above the liquid, plus the static fluid pressure $\rho gh$

$$P_h = P_0 + \rho gh$$

This means that the pressure is the same at all points on a horizontal line in a fluid (all of these points are at the same depth).

Archimedes Principle: a fluid exerts an upward buoyant force $F_B$ on an object in the fluid

- Magnitude of $F_B$ equal to the weight of the fluid displaced by an object

$$F_B = \frac{m_{\text{displaced}}}{\text{fluid}} \cdot g$$

$$F_B = (\rho_{\text{fluid}} \cdot V_{\text{displaced}}) \cdot g$$

$V_{\text{displaced}}$ is the volume that is actually under water.
0. Reflections on Last Assignment (1 pt)

Pick one question from Homework 4 that you found particularly difficult and

a) describe any mistakes or misunderstandings you made

b) describe the best strategies to ensure you learn from your mistakes and won’t have the same misunderstanding again

1. Let it Fall (1 pt) Explicitly show that an object of density $\rho$ will sink when submerged in a fluid of density $\rho_{\text{fluid}}$, where $\rho_{\text{fluid}} < \rho$, and find its acceleration. Assume the impact of the drag force is negligible.

2. Hanging by a Thread (2 pts). A spider spins a fine thread of spider silk between two posts a distance $L_0$ apart. The thread has a diameter of $d = 5 \times 10^{-6}$ m and an equilibrium length of $L_0$. When the spider (with mass $m = 2$ milligram) hangs from the center of the thread, the thread sags and stretches to support the weight of the spider, making an angle of $\theta = 10^\circ$ below the horizontal, as shown.

a) Why must the thread sag when the spider hangs from it?

b) Calculate the tensile strain $\varepsilon$ in the thread and the Young’s modulus $Y$ for the silk.

3. Under pressure (3 pts) Your blood pressure (typically reported as “120 over 80” or something like that) has two parts: the first number is the systolic pressure, which is the arterial pressure during the contraction of the heart, and the second number is the diastolic pressure, which is the arterial pressure when the heart is relaxed.

a) Blood pressures are typically given in torr, or millimeters of mercury (1 torr = 1 mmHg). Using the fact that mercury has a density of 13.6 g/cm$^3$, derive a conversion factor between torr and pascals, and use it to convert 120 torr into Pa.

b) These blood pressures are known as gauge pressures, meaning that they are the amount by which the blood pressure exceeds atmospheric pressure. The first measurement of blood pressure involved inserting a long cannula (needle-tipped tube) into the artery of a horse. The other end of the tube was left open to the atmosphere. If the horse’s systolic blood pressure were 150 mmHg, how high would the blood from that artery have risen in the cannula? Assume that blood has the same density as water.
c) As we’ll see when we study fluid dynamics, fluids flow from high pressure to low pressure. Using your results from parts a) and b), why must IV bags be placed at least 20 cm above a patient’s arm?

4. Head over heels (3 pts) Blood pressures are measured at the height of the heart. If the maximum (systolic) pressure at the brain drops below zero (gauge pressure), then no blood will reach the brain. This condition can lead to loss of consciousness. If the systolic pressure at the brain exceeds zero but the diastolic pressure does not, blood will reach the brain sporadically instead of continuously. This can cause dizziness, but usually not blackout.

a) What is the minimum pressure (measured at the heart) needed to avoid dizziness? Is this the minimum systolic or diastolic pressure?

b) A giraffe’s head is about 3 meters above its heart. The giraffe’s circulatory system must be specially adapted for its unusual anatomy. What must be the minimum diastolic pressure at the heart of a giraffe? If the systolic:diastolic ratio is the same for giraffes as for humans, what is the systolic pressure of a giraffe at the level of its heart?

c) Conversely, the blood pressure in your feet is greater than the pressure at your heart. Estimate the maximum (systolic) blood pressure in your feet, if the systolic pressure at your heart is 120 mmHg. This increased pressure can lead to swelling of the feet.

d) Estimate the maximum (systolic) blood pressure in giraffe’s feet, which are 2.5 meters below its heart. Giraffes have very tight elastic skin around their ankles to support this pressure and prevent swelling.