Physical Sciences 2

Exam 1 (Individual)
Tuesday, October 2, 2018

Your name: ____________________________

Section TF: ____________________________

**Do not** turn the page until you are told to begin. You will be given 75 minutes to complete this exam. Show all your work on the exam itself; no credit will be given for anything written on other paper. Please box your final answer to each calculation.

You may use a calculator if you have brought one. You may refer to one 8.5”x11” sheet of notes, which must be in your own handwriting. Turn in your notes along with the exam when time is called.

This exam contains 5 sheets of paper (including this one), with 5 problems.

Do not write in the following table; it will be used for grading.

| Problem 1  | ___ / 25 |
| Problem 2  | ___ / 20 |
| Problem 3  | ___ / 16 |
| Problem 4  | ___ / 19 |
| Problem 5  | ___ / 20 |
| **Total**  | ___ / 100 |
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Problem 1: Multiple Choice [25 points]

For each of the following questions, circle the letter(s) corresponding to the best answer(s) from the options given. Each question is worth 5 points; partial credit will be given for those problems asking you to “Circle all that apply.”

a) A 30-kg boy and a 60-kg man are both initially at rest and standing 10.0 m apart on a frictionless surface. They each hold one end of a massless rope. The boy then pulls on the rope so that he (the boy) moves 2.0 m relative to the ground, in the direction of the man. Which of the following statements is true? **Circle ALL that apply.**

A) The man will not move.
B) Their CM will move 1.0 m in the direction of the man.
C) Their CM will not move.
D) The man will move 2.0 m in the direction of the boy.
E) The man will move 1.0 m in the direction of the boy.

b) A spaceship is moving in deep space, far from any stars. When must it fire its rockets? (this particular spaceship is capable of firing rockets in any direction) **Circle ALL that apply.**

A) To continue moving at a constant speed.
B) To speed up.
C) To slow down.
D) To change direction.
E) To move along a circular path.

c) A softball player catches a ball of mass, m, which is moving toward her with a horizontal speed, v. While bringing the ball to rest, her hand moves back a distance, d. Assuming a constant deceleration, what is the magnitude of the horizontal force exerted on the ball by her hand?

A) \( \frac{mv^2}{2d} \)
B) \( \frac{mv^2}{d} \)
C) mvd
D) \( 2mv/d \)
E) \( mv/d \)

continued...
d) A pilot drops a supply package from a plane flying horizontally. Assuming that the plane doesn’t change speed or direction and neglecting air resistance, when the package hits the ground, the horizontal location of the plane will:

A) be behind the package.

\(\text{B) be over the package.}\)

C) be in front of the package.

D) depend on the speed of the plane when the package was released.

e) You are helping a friend move into a new apartment. A box of mass \(m\) needs to be moved. You are taller than the box, so you reach down to push it an \textit{angle of 15 degrees} with respect to the horizontal. You continuously push harder and harder until the moment it begins to move, at which point you continue to push with that exact same force (angle and direction). That is, you maintain the \textit{maximum} pushing force, applied right before it began to move. Assume the following about the friction coefficients: \(\mu_s > \mu_k > 0\). Which of the following statements is true? \textbf{Circle ALL that apply.} (hint: think carefully about the magnitude of the normal force)

A) Once the box begins to move, it will continue at a constant speed.

B) Before the box begins to move, the \textit{magnitude} of the static friction force is equal to the \textit{magnitude} of the pushing force.

C) Once you’ve begun pushing, but before the box begins to move, the static friction force is constant.

D) Once the box begins to move, the frictional force has magnitude \(mg\mu_k\).

\(\text{E) The magnitude of the static friction force will exceed } mg\mu_s \text{ at some point during the pushing.}\)
Problem 2: Lift Off! [20 Points]

A rocket blasts off from its launch pad at Kennedy Space Center with a constant vertical acceleration of 100 m/s\(^2\). It maintains this acceleration for a time interval of exactly one minute, after which its engines burn out and the rocket acceleration becomes \(g\), the acceleration due to gravity. Neglect air resistance and assume that \(g\) is constant.

a) How high above the ground is the rocket at the instant its engines burn out?

\[
\begin{align*}
h(t) &= h_0 + v_{0y}t - \frac{1}{2}a_y t^2 \\
&= \frac{1}{2} (100 \text{ m/s}^2) \cdot (60 \text{ s})^2 \\
&= 180,000 \text{ m}.
\end{align*}
\]

b) What is the speed of the rocket at the instant its engines burn out?

\[
\begin{align*}
v_{f,y} &= v_{0y} + a_y t \\
v_{f,y} &= 100 \text{ m/s} \cdot 60 \text{ s} = 6000 \text{ m/s}
\end{align*}
\]

continued...
c) How long (in seconds) will the rocket continue to rise after its engines have burned out?

\[ v_{i, y} = 0 \text{ ms}^{-1} \]

\[ v_{f, y} = v_{i, y} + ay \cdot t \]

\[ \frac{v_{f, y} - v_{i, y}}{ay} = t = \frac{0 \text{ ms}^{-1} - 6000 \text{ ms}^{-1}}{-9.81 \text{ ms}^{-2}} = \text{cell. 0.023s} \]

\[ 6000 \text{ ms}^{-1} = \text{cell. 0.023s} \]

\[ \frac{6000 \text{ ms}^{-1}}{-9.81 \text{ ms}^{-2}} = \text{cell. 0.023s} \]

\[ d) \text{ What is the maximum height of the rocket above the ground?} \]

\[ h_{\text{total}} = h_{\text{rockets}} + h_{\text{no rockets}} \]

\[ 180,000 \text{ m} \]

\[ (\text{from part A}) \]

\[ h_f = h_i + v_{i, y} \cdot t + \frac{1}{2}ay \cdot t^2 \]

\[ h_f = 180,000 \text{ m} + 6000 \text{ ms}^{-1} \cdot \text{cell. 0.023s} + \frac{1}{2}(-9.81 \text{ ms}^{-2}) \cdot (\text{cell. 0.023s})^2 \]

\[ = 2,014,862.4 \text{ m} \]

\[ V_f^2 - V_i^2 = 2a \cdot h_{\text{no rockets}} \]

\[ \frac{V_f^2 - V_i^2}{2a} = h_{\text{no rockets}} = \frac{(6000 \text{ ms}^{-1})^2}{2 \cdot (-9.81 \text{ ms}^{-2})} = 1,834,862.4 \text{ m} \]

\[ h_f = h_{\text{rockets}} + h_{\text{no rockets}} = 2,014,862.4 \text{ m} \]
Problem 3: Figure Skating [16 points]

A couple is figure skating in a pair (assume no friction). He weighs 60 kg, and she weighs 40 kg. He’s originally skating due east, carrying her in his arms, and skating at 2 m/s. Then she leaps from his arms. After she jumps, she is traveling (with respect to the ice) at 4 m/s southeast (i.e. 45° south of east).

What is his speed and direction after she jumps? (Express his direction in terms of the number of degrees from the east.)

\[ \vec{F}_x = \vec{F}_y \]

\[ \vec{F}_{ix} = \vec{F}_{iy} \]

\[ (m_1 + m_2) \cdot \vec{v}_{ix} = m_1 \cdot \vec{v}_{ix} + m_2 \cdot \vec{v}_{ix} \]

\[ (m_1 + m_2) \cdot \vec{v}_{ix} = (100 \text{kg}) \cdot 2 \text{m/s} - (60 \text{kg} \cdot 10 \text{m/s} \cdot \cos(45°)) \]

\[ \vec{v}_{ix} = 1.45 \text{m/s} \]

\[ \vec{v}_{iy} = \vec{v}_{iy} \]

\[ (m_1 + m_2) \cdot \vec{v}_{iy} = m_1 \cdot \vec{v}_{iy} + m_2 \cdot \vec{v}_{iy} \]

\[ -m_2 \cdot \vec{v}_{iy} = \vec{v}_{iy} = -\frac{40 \text{kg} \cdot 10 \text{m/s} \cdot \sin(45°)}{60 \text{kg}} = 1.87 \text{m/s} \]

\[ \text{Speed} = \sqrt{\vec{v}_{ix}^2 + \vec{v}_{iy}^2} \]

\[ \text{Direction} = \theta = \tan^{-1} \left( \frac{1.87 \text{m/s}}{1.45 \text{m/s}} \right) \]

\[ \theta = 52.5° \]

\[ = 2.38 \text{m/s} \]
Problem 4: You spin me right round [19 points]

Consider a system made up of three objects glued together: a solid uniform disk of mass 6.0 kg with a diameter of 2.0 meters, and two small circular masses each with a mass of 2.0 kg and fixed at various angles along the rim of the disk, as shown in the figure to the right. The origin of a reference coordinate system is placed at the center of the large disk.

a) Find the \((x, y)\) coordinates of the center of the mass of this system. \(\text{(hint: don't forget that the large disk also has mass, and you can do your calculation imagining that its mass is concentrated at its center)\}

\[
\begin{align*}
R &= 2.0\text{m} \Rightarrow R = 1.0\text{m}. \\
\text{cm}_x &= \frac{1}{M} \left( M_1 \cdot r_1 \cdot x + M_2 \cdot r_2 \cdot x + M_3 \cdot r_3 \cdot x \right) \\
&= \frac{1}{10\text{kg}} \left( 6.0\text{kg} \cdot 1.0\text{m} \cdot \cos(10^\circ) + 2.0\text{kg} \cdot 1.0\text{m} \cdot \sin(10^\circ) \right) = 0.1185\text{m}. \\
\text{cm}_y &= \frac{1}{M} \left( M_1 \cdot r_1 \cdot y + M_2 \cdot r_2 \cdot y + M_3 \cdot r_3 \cdot y \right) \\
&= \frac{1}{10\text{kg}} \left( 6.0\text{kg} \cdot 1.0\text{m} \cdot \sin(10^\circ) + 2.0\text{kg} \cdot 1.0\text{m} \cdot \cos(10^\circ) \right) \\
&= 0.3255\text{m}.
\end{align*}
\]

b) The entire system is rotated one full turn about the origin.

What distance does the center of mass travel in one full rotation?

\[
\text{Radial Distance} = R = \sqrt{R_{cm,x}^2 + R_{cm,y}^2} = \sqrt{(0.1185\text{m})^2 + (0.3255\text{m})^2} = 0.3464\text{m}.
\]

\[
\theta = 2\pi \cdot R = 2\pi \cdot 0.3464\text{m} = 2.17\text{rad}
\]

What displacement does the center of mass undergo in one full rotation?

\[
\text{Displacement} = 0\\text{m} \quad \text{BYC initial position = final position (since 1 full rotation)}
\]

continued...
c) You wish to balance the system by shifting its center of mass directly to the origin. In order to accomplish this, you are able to place a small, yet more massive, 4.0 kg circular mass anywhere on the disk. Where should you place the 3rd circular mass such that the new center of mass for the system will be precisely at the origin?

\[
\begin{align*}
C_{M,x} &= \frac{1}{M_T} \left( M_1 \cdot R_{1,x} + M_2 \cdot R_{2,x} + M_3 \cdot R_{3,x} + M_4 \cdot R_{4,x} \right) \\
C_{M,y} &= \frac{1}{M_T} \left( M_2 \cdot R_{2,y} + M_3 \cdot R_{3,y} + M_4 \cdot R_{4,y} \right) \\
C_{M} &= \frac{M_2 \cdot R_{2,y} + M_3 \cdot R_{3,y} + M_4 \cdot R_{4,y}}{M_T} \\
C_{M,x} &= \frac{M_1 \cdot R_{1,x} + M_2 \cdot R_{2,x} + M_3 \cdot R_{3,x} + M_4 \cdot R_{4,x}}{M_T} \\
\Rightarrow R_{4,x} &= \left( \frac{M_2 \cdot R_{2,x} + M_3 \cdot R_{3,x}}{M_4} \right) = \left( \frac{2.0 \text{kg} \cdot 1.0 \text{m} \cdot \cos(45^\circ) + 2.0 \text{kg} \cdot 1.0 \text{m} \cdot -30^\circ}{4 \text{kg}} \right) \\
R_{4,x} &= -0.2962 \text{m} \\

\text{by same logic:} \\
R_{4,y} &= \left( \frac{M_2 \cdot R_{2,y} + M_3 \cdot R_{3,y}}{M_4} \right) = \left( \frac{2.0 \text{kg} \cdot 1.0 \text{m} \cdot \sin(45^\circ) + 2.0 \text{kg} \cdot 1.0 \text{m} \cdot \cos(10^\circ)}{4 \text{kg}} \right) \\
R_{4,y} &= -0.8138 \text{m} \\
\hat{R}_{4} &= (-0.2962 \text{m}, -0.8138 \text{m})
\end{align*}
\]
Problem 5: Winter vacation [20 points]

A skier of mass $M$ starts from rest at the top of a mountain, points her skis down the trail, and begins to glide. Assume that the coefficient of kinetic friction between the snow and skis is $\mu_k$.

Ignoring air drag, how long will it take her to finish a straight run of distance $L$ that makes an angle $\theta$ with the horizontal? Your answer could depend on some or all of the following quantities: $M$, $\mu_k$, $g$, $L$, and $\theta$. Be sure to draw a clear, labeled free-body diagram as part of your answer.

\[ \sum F_y = F_d - M g \cos(\theta) = M a_y = 0 \]
\[ \Rightarrow F_d = M g \cos(\theta) \]

\[ \sum F_x = M g \sin(\theta) - F_{kx} = M g \sin(\theta) - F_d \mu_k = M a_x \]
\[ = M g \sin(\theta) - M \mu_k g \cos(\theta) = M a_x \]
\[ \Rightarrow a_x = g \sin(\theta) - \mu_k g \cos(\theta) \]

$$d_t = d_0 + V_{0x} t + \frac{1}{2} a_x t^2$$

$$d_t = L; \quad d_0 = 0; \quad V_{0x} = 0 \text{ m/s}$$

$$L = \frac{1}{2} a_x t^2 \quad \Rightarrow \quad t^2 = \frac{2L}{a_x} \quad \Rightarrow \quad t = \sqrt{\frac{2L}{a_x}}$$

$$t = \sqrt{\frac{2L}{g \sin(\theta) - \mu_k g \cos(\theta)}}$$