Pre-reading for Lecture 4b: Work and Kinetic Energy

We’ve seen two distinct ways of solving physics problems: we can work out all the forces and motion (using FANCLAN or something like that), or we can use the conservation of energy. In many cases either approach is OK, although using energy is often much easier (when it is applicable).

One clue that you cannot use energy is if the question involves the time required to do something. Energy conservation tells you about the change between two states—it doesn’t tell you how long it takes for the change to take place.

This lecture will introduce the concept of work. “Work” is the connection between force and energy. In particular, we will show that:

The total work done on an object equals the change in that object’s kinetic energy.

This is a deep and fundamental relationship that can be expressed mathematically:

$$\Delta K_{\text{CM}} = \int_{\text{initial}}^{\text{final}} \vec{F}_{\text{net}} \cdot d\vec{r}_{\text{CM}}$$

The left hand side represents the change in the kinetic energy of the center-of-mass motion of the object. The right hand side is an integral of the net force “dot” the infinitesimal displacement $d\vec{r}_{\text{CM}}$ of the center of mass. That integral on the right hand side is what we call the total work or net work done by all forces on the object.

For instance, if the person pulling the sled in the figure above exerts a constant force $\vec{F}$ at an angle $\theta$ over a distance $D$, the work done by that person on the sled is $FD\cos\theta$.

Note that the “$\cos \theta$” term in the example of the sled comes from the dot product! You do recall the dot product, right? You also see that the work done by the person is positive: this means that the pulling force, acting alone, would increase the kinetic energy of the sled. That makes sense, right?

Thus work is directly related to kinetic energy: a force that does positive work, acting alone, would increase the kinetic energy of an object (i.e. speed it up), while a force that does negative work, acting alone, would decrease the kinetic energy (i.e. slow it down). A force that does zero work, acting alone, would not change the kinetic energy.

For instance, if a wagon is rolling down a hill, the force of gravity is doing positive work (making the wagon go faster), but the force of air drag is doing negative work (making the wagon go slower). The normal force of the road on the wagon does no work, because it always acts perpendicular to the displacement of the wagon, so its dot product is zero.

Using the concept of work, you can solve many problems using energy conservation in a simple manner that would be much more complicated using forces and FANCLAN. This is the main reason why we introduce the concept of work: it allows us to use information about energy to calculate things related to force.
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For instance, if a car crashes into a wall and comes to rest, you can estimate the force of the impact by calculating the change in kinetic energy of the car, and the displacement of the car’s center of mass during the impact. This is often easier than any other method.

• Learning objectives: After this lecture, you will be able to…

1. Identify whether a problem would best be solved by using forces or by using energy.
2. Define the center-of-mass work done by a force on an object.
3. Calculate the center-of-mass work for a constant force (like gravity) on an object.
4. Identify whether the work done by a force on an object is positive, negative, or zero.
5. Use the center-of-mass equation to calculate the work done by one force if the work done by other forces is known.
Activity 1: Force and Energy

1. A block of mass $m$ is sliding without friction on a plane inclined at an angle $\theta$ from the vertical (see figure). The block has some initial kinetic energy $K_i$.

   You later notice that the block has slid down the plane by a distance $L$ from its initial position. It now has some new kinetic energy $K_f$. Find an expression for the change in kinetic energy, $K_f - K_i$, in terms of the mass $m$, the distance $L$, the angle $\theta$, and any relevant physical constants. (Hint: what would be the easiest way to do this?)

2. Recall that the displacement vector, $\Delta \vec{r}$, is defined as $\vec{r}_f - \vec{r}_i$. Show that the change in kinetic energy $\Delta K$ that you calculated in part (1) is related to the gravitational force $\vec{F}_g$ by:

   $$\Delta K = \vec{F}_g \cdot \Delta \vec{r}$$

   (yes, that is the dot product!)

Bonus! How much time elapses between the initial and final states of the block? (Think carefully about this one…)
Activity 2: Calculating work

• We define the center-of-mass work (or just “work”) done by a force $F$ on an object as:

$$W_{by F} = \int_{\text{initial}}^{\text{final}} \vec{F} \cdot d\vec{r}$$

This is the gold standard definition of work, at least as far as this course is concerned. Note that $d\vec{r}$ is the infinitesimal displacement of the center of mass of the object.

1. You pull a sled using a constant force $\vec{F}$ at an angle $\theta$ from the horizontal. The sled travels a distance $D$. Find an expression for the work done by you on the sled.

• Bonus! A roller coaster rolls on a curving track. Show that the normal force of the track on the roller coaster does no work.
Activity 3: Positive, Negative, or Zero?

- Under what circumstances will the work done by a force be positive?
- Under what circumstances will the work done by a force be negative?
- Under what circumstances will the work done by a force be zero?

2. In each of the following scenarios, circle whether the work is positive, negative, or zero:

- The force of air drag on a car speeding down the highway.
  Work is: positive negative zero

- The force of gravity on an object falling through the air.
  Work is: positive negative zero

- The force of gravity on a rocket being launched up into the air.
  Work is: positive negative zero

- The normal force of the road on a car while the car is driving.
  Work is: positive negative zero

- The force you exert to hold a heavy box motionless over your head.
  Work is: positive negative zero

• Bonus! Find an example in which the static friction force does positive work…
Activity 4: Putting it Together: Work

1. While skiing, you fall from a (100 foot; 30m) cliff, reaching a speed of 25 m/s (56 mph), and land in deep snow, forming a crater 2 meters deep. Estimate the force exerted on you by the snow during landing. (Take your mass to be \( m = 70 \text{ kg} \), and assume the force is constant). Solve this question using work and energy!

• Bonus! Show that the work done by gravity on any object is \( W_{\text{by gravity}} = -mg\Delta h \), where \( \Delta h \) is the change in the height of the object.
One-Minute Paper

Your name: _____________________________       TF: _____________________________

Names of your group members:  _________________________________
_________________________________
_________________________________

• Please tell us any questions that came up for you today during lecture. Write “nothing” if no questions(s) came up for you during class.

• What single topic left you most confused after today’s class?

• Any other comments or reflections on today’s class?