Physical Sciences 2: Assignment for September 25 – October 11
Homework #4: Energy, Force, and Work
Due Thursday, October 11, at 9:00 AM

This assignment must be turned in by 9:00AM on Thursday, October 11. Late homework will not be accepted. Please write your answers to these questions on a separate sheet of paper with your name and your section TF’s name written at the top. Turn in your homework to the basket in the front of lecture with your section TF’s name.

You are encouraged to work with your classmates on these assignments, but please write the names of all your study group members on your homework.

After completing this homework, you should…
- Know how to calculate the kinetic energy of an object.
- Know the units associated with energy.
- Know how to calculate the gravitational potential energy of an object.
- Be able to calculate efficiency when energy changes forms.
- Know how to calculate elastic potential energy.
- Know how energy changes during the movement of an oscillator.
- Know the principle of conservation of energy and how to apply it.
- Be able to calculate power.
- Be able to interpret a potential energy vs. time graph.
- Be able to combine the principles of conservation of energy and conservation of momentum in order to solve for various unknown quantities in problems.
- Be able to calculate work done from information about forces and distances.
- Understand and be able to use the work-energy theorem.
- Be able to interpret a force vs. distance diagram.
Here are summaries of this lecture’s important concepts to help you complete this homework:

Module 4: Energy, Force, and Work
Compiled by Kristina Callaghan

**Energy**
- For an isolated system, energy is conserved: energy is neither created nor destroyed but only changes forms
  \[ \Delta E_{\text{total}} = 0 \]
- The total energy of a system is the sum of its mechanical and internal energy
  \[ E_{\text{total}} = E_{\text{mech}} + E_{\text{internal}} \]
- Mechanical energy \( E_{\text{mech}} \) is the sum of the kinetic and potential energy of the system
  \[ E_{\text{mech}} = K + U \]
- Internal energy \( E_{\text{internal}} \) accounts for the other forms of energy (i.e., besides kinetic and potential) that may be present in the system, such as: light, sound, chemical energy, thermal energy, nuclear energy, etc.
  - Often assume \( E_{\text{internal}} \) does not change

**Work**
- The total work done on an object is equal to the change in the object’s kinetic energy
  \[ W_{\text{net}} = \Delta K \]
- Work is also equal to
  \[ W_{\text{net}} = \int F_{\text{net}} \cdot dr \]
- Positive work increases an object’s kinetic energy
- Negative work decreases an object’s kinetic energy
- Zero work does not change an object’s kinetic energy
Forces
- conservative forces arise from the potential energy of an interaction and are path-independent
  - path-independent means that the force affects the object in the same way no matter what path the object takes
  - examples: gravity, spring force
- nonconservative forces have no associated potential energy and are path-dependent
  - path-dependent means that how the force affects an object depends on the path the object takes
- relationship to work and energy:
  - work done by conservative forces equals the change in potential energy
    \[ W_{\text{by conservative force}} = -\Delta U \]
  - work done by nonconservative forces equals the change in mechanical energy
    \[ W_{\text{net}} = \Delta K \]
\[ W_{\text{cons}} + W_{\text{noncons}} = \Delta K \]
\[ -\Delta U + W_{\text{noncons}} = \Delta K \]
\[ W_{\text{noncons}} = \Delta K + \Delta U = \Delta E_{\text{mech}} \]
\[ \Rightarrow \text{for an isolated system, } W_{\text{noncons}} \text{ also equals the negative change in internal energy} \]
\[ \Delta E_{\text{total}} = \Delta E_{\text{mech}} + \Delta E_{\text{internal}} = 0 \]
\[ \Delta E_{\text{mech}} = -\Delta E_{\text{internal}} \]
\[ W_{\text{noncons}} = -\Delta E_{\text{internal}} \]

Power
- power is the rate of doing work
\[ P = \frac{\Delta E}{\Delta t} \]
\[ P_{\text{noncons}} = \frac{dW}{dt} = F \cdot \dot{v} \]
0. Reflections on Last Assignment (0 pt)

Pick one question from Homework 3 that you found particularly difficult and:

a) describe the errors that you made

b) ways to ensure that you have learned from your mistakes so that you won’t have the same trouble with such a question in the future

1. Free-fall revisited (2 pt)

Consider an object in free-fall that is moving only in the vertical direction. Ignore drag or friction. Write a differential equation for the mechanical energy of this object in terms of its position (y). In order for mechanical energy to be conserved, this equation must always equal a constant.

Assume that the position of the object can be described by a power series in time, including up to a cubic term:

\[ y(t) = A + Bt + Ct^2 + Dt^3 \]

a) Show explicitly that energy conservation requires that \( D = 0 \) and \( C = -1/2g \), but that \( A \) and \( B \) can take on any value.

b) Show that \( A \) is equal to the position of the object when \( t = 0 \), and that \( B \) is equal to the y-component of the velocity when \( t = 0 \).
2. Energy Conversion (1 pt)

For each of the following processes, describe what kind of energy is being converted into what other kind(s). Valid forms of energy for the purpose of this question are “kinetic energy of (object),” “gravitational potential energy,” “elastic potential energy of (object),” or “internal energy of (object).” There may be more than one conversion involved, so your answer should be a kind of narration of the energy conversion(s).

For example, if the process is, “A girl skating on rollerblades skids to a halt,” then your answer should read something like:

The kinetic energy of the girl is converted into internal energy of the skates and of the road.

• You drop a book, and it falls to the floor and lands with a thud.

• You drop a rubber ball, which hits the floor and rebounds back up to your hand. (Hint: Think about what happens to the ball while it is in contact with the floor.)

• You pour milk from a carton into a drinking glass.

• You pick up a heavy box from the floor and lift it onto a table.

• You stretch a rubber band between your thumb and forefinger and release the rubber band. The rubber band shoots straight up into the air, falls back down, and lands motionless on the ground.

3. Running up the stairs (3 pts). Time how long it takes for you (or a friend) to run as fast as possible up a staircase. Measure the height of one step and count the steps to find how high you climbed.

a) How much mechanical energy did you produce to climb the stairs?

b) What was the average mechanical power produced during your climb?

c) Estimate the muscle mass in your upper legs (quadriceps, hamstrings, and buttocks). (Recall that biological tissue has about the same density as water.) If each kilogram of muscle can produce about 100 watts of mechanical power, are these muscles sufficient to provide the power required for you to run up the stairs?

d) If you are 25% efficient at converting food energy to mechanical energy, and there are about 4200 joules in one food Calorie, how many Calories did you consume during your climb?

e) The food energy that was not converted into mechanical energy was mainly turned into heat. If your body were completely insulated (for instance, if you were wearing heavy winter clothes), how much would your body temperature increase as a result of running up the stairs? Assume that your entire body has a specific heat equal to that of water, $C = 4.18 \, \text{J/g} \cdot {^\circ\text{C}}$. 
4. Muscle up (2 pts). Muscle motion is caused by the relative motion of actin and myosin filaments inside the muscle fibers, as shown in the following diagram:

When your muscles are activated, the myosin molecules “crawl” along the actin filaments, pulling them together and shortening the entire muscle fiber. As with the case of kinesin, the myosin takes discrete “steps” along the actin filaments.

a) In one “step,” a myosin molecule pulls the actin a distance of about 10 nm with a force of about 3 pN. What is the magnitude of the work done by a single myosin molecule in a single step?

b) The hydrolysis of one mole of ATP provides 31 kJ of energy. Is it possible that one molecule of ATP can provide the energy required for one myosin molecule to take a single step? Support your answer with a calculation.

c) Conversion of glucose into ATP is only about 50% energy-efficient. What is the overall efficiency for converting glucose energy into mechanical work by your muscles? (Note that this calculation ignores the fact that your body consumes energy even when it is entirely at rest.)
5. Putting Everything Together: Block on Ramp (2 pts)

a) A block is launched up a frictionless ramp, as shown, with initial speed $v_i$. The block travels up the ramp and continues across the level section.

i. Which forces, if any, do non-zero work on the block?

ii. Which forces, if any, do zero work on the block?

b) Suppose the block in part a is launched with the same initial speed $v_i$ on the following frictionless ramps. In each case, state whether the magnitude of the net work done on the block from the bottom to the top of the ramp is greater than, less than, or equal to the magnitude of the net work done on the block in part A. Explain your answer in each case.

i. The ramp is steeper ($\alpha > \theta$).

ii. The ramp has two sections of different slope.

iii. The ramp has several sections of gradually increasing steepness.

iv. The ramp is curved.