Pre-Reading for Lecture 3b: Friction and Drag

As you know, whenever two solid objects are in contact there is a "pushing force" (normal force) between the objects that acts perpendicular to the plane of contact between the objects. In many circumstances, there is also a force that acts parallel to the plane of contact between two objects. This force is called a friction force.

There are two different kinds of friction force:

- If the objects are not sliding with respect to one another, then the force of static friction will act to prevent them from sliding. The static friction force has a maximum magnitude given by $\mu_kF_n$, where $\mu_k$ is a constant that depends on the objects and $F_n$ is the magnitude of the normal force between the objects. To an excellent approximation, the static friction force does not depend on the area of contact between the objects.

- If the objects are sliding with respect to one another, then the force of kinetic friction will act to oppose the relative sliding motion. The kinetic friction force has a magnitude equal to $\mu_kF_n$, where $\mu_k$ is a constant that depends on the objects and $F_n$ is the magnitude of the normal force between the objects. To an excellent approximation, the static friction force does not depend on the area of contact or the speed of sliding between the objects. The kinetic friction force acts in whatever direction will tend to reduce the relative sliding motion between the objects.

When a solid object moves with respect to a fluid, there is a drag force that tends to oppose the relative motion of the object and the fluid. Drag forces are conceptually similar to friction forces, but they do depend on the speed of the object (higher speeds give bigger drag forces), and on the size of the object (bigger objects have bigger drag forces). They can also depend on the shape of the object, which is why many objects are "streamlined" to minimize drag forces in the air (a sports car) or in water (a porpoise).

There are two different kinds of drag force:
- For small objects moving slowly through viscous fluids, the main force is viscous drag, which arises because the moving object literally has to drag fluid molecules along with it as it moves. You can think of viscous drag as acting along the “sides” of the moving object; think of a knife being pulled out of a jar of honey. Viscous drag is the main drag force acting on microscopic objects such as cells and organelles. For a spherical object, the magnitude of the viscous drag force is \( F_{\text{drag}} = 6\pi \eta Rv \), where \( \eta \) is the viscosity, \( R \) is the radius, and \( v \) is the speed. For a given volume, the smallest viscous drag can be achieved using a spherical shape.

- For large objects moving quickly through non-viscous fluids, the main force is pressure drag, which arises because the moving object has a higher pressure in front of it than behind it. You can think of pressure drag as acting on the “face” of the object as it pushes the fluid away; think of the wind in your face as you ride a bicycle. Pressure drag is the main drag force acting on macroscopic objects such as people, birds, fish, etc. The magnitude of pressure drag is given by \( F_{\text{drag}} = \frac{1}{2} C_d \rho A v^2 \), where \( \rho \) is the density of the fluid, \( A \) is the cross-sectional area of the object, \( v \) is the speed, and \( C_d \) is a dimensionless quantity known as the “drag coefficient” that depends on the shape of the object and also (to some extent) on its size and speed. For a given volume, the smallest pressure drag can be achieved by “streamlining” the object to minimize cross-sectional area.

Learning objectives: After this lecture, you will be able to…

1. Describe and compare the two kinds of friction force that act between solid objects: static friction and kinetic friction.

2. Explain the force laws for these two kinds of friction forces, and describe what happens when the friction force changes from static to kinetic (or vice versa).

3. Use the force laws for static and kinetic friction to solve problems in Newtonian dynamics.

4. Describe and compare the two kinds of drag forces that a fluid can exert on an object: viscous drag and pressure drag. Explain how these forces arise from interactions between the object and the fluid.

5. Identify in a given situation whether viscous drag or pressure drag will predominate.

6. Use the force laws for viscous drag and pressure drag to solve problems in Newtonian dynamics.
Activity 1: To slide or not to slide

- Consider the following scenario: An object of mass $M$ is observed to remain at rest on an inclined ramp (at an angle $\theta$). Do a complete Newtonian dynamics analysis (use the FANCLAN checklist!). Show that there must be a contact force of the ramp on the object that acts parallel to the plane of contact. What is the magnitude and direction of this contact force?

![Diagram of an object on an inclined plane with forces labeled F_x, F_y, and F_N.]

**Find magnitude of $F_x$ in terms of $m, g,$ and $\theta$**

1. $F_x = m\alpha_x \Rightarrow -F_x + F_y \sin \theta = ma_x$
2. $F_y = m\alpha_y \Rightarrow F_N - F_y \cos \theta = ma_y$

- $\alpha = (0, 0)$
- $F_y = mg$
- $F_N - F_y \sin \theta = 0$
- $F_x = mg \sin \theta$

**Bonus!** The static friction force has a maximum magnitude equal to $\mu_s F_N$, where $\mu_s$ is a dimensionless constant and $F_N$ is the magnitude of the normal force. If the object just begins to slide at an angle $\theta_{\text{max}}$, find an expression for $\mu_s$.

$$M_s F_N = mg \sin \theta \Rightarrow M_s mg \cos \theta = mg \sin \theta$$

$$\Rightarrow M_s = \tan \theta,$$ where $\theta = \theta_{\text{max}}$
Static vs. Kinetic Friction (Pre-Video)

- What is static friction?
  - If there's no sliding between solid objects, static friction prevents sliding.
  - Is there a force law for static friction?
    - $F_{s,F} \leq \mu_s F_N$ normal force
    - magnitude; acts in direction needed to prevent sliding

- What is kinetic friction?
  - If there's sliding between objects, kinetic friction acts to oppose sliding motion.
  - Is there a force law for kinetic friction?
    - $F_{k,F} = \mu_k F_N$
    - magnitude

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![Diagram of friction forces]

- $f_{s,max} = \mu_s N$
- $F_{pull} > F_{k,F} \rightarrow$ Sliding
- $F_{friction} \rightarrow F_{pull}$ (Applied)

F, Applied Force
Activity 2: Kinetic Friction

I'm going to pull a sled (with some people on it) across the room. If I pull the sled at constant velocity, what force must I exert to pull the sled? (Use the FANCLAN checklist!) Express your answer in terms of the kinetic friction coefficient $\mu_k$, the mass of the sled (and people) $m$, and any relevant physical constants. Assume that I'm pulling horizontally on the rope attached to the sled.

\[
\begin{align*}
\sum F_x &= m a_x \Rightarrow F_{\text{pull}} - F_{k_f} = m a_x \\
\sum F_y &= m a_y \Rightarrow F_n - F_g = m a_y
\end{align*}
\]

\[\vec{a} = (0, 0)\]

\[F_g = mg \quad F_{k_f} = m_k F_n\]

\[F_{\text{pull}} = m_k F_n\]

\[F_n = mg\]

\[m \uparrow \quad F_{\text{pull}} \uparrow \quad \text{Tested!}\]

**Bonus!** How would your answer change if I pulled at an angle $\theta$ above the horizontal?
When an object moves in a fluid (either liquid or gas), the fluid will exert forces on the object. These forces can be quite complicated!

We’ll start by discussing the simplest kind of forces due to moving fluids: **drag forces**. Drag forces can depend on:

- the **speed** of the object (with respect to the fluid)
  
  \[
  \text{Faster} \rightarrow \text{bigger drag}
  \]

- the **size** of the object
  
  \[
  \text{Larger} \rightarrow \text{bigger drag}
  \]

- and the **shape** of the object
Two kinds of drag forces

- There are two distinct "mechanisms" by which fluids can exert drag forces: either through viscosity or through pressure. What is the difference?

Viscous drag
- dragging
- low pressure fluid along

Pressure drag
- high pressure

- Viscous drag depends on the viscosity of the fluid, \( \eta \), which has SI units of \( \text{kg m}^{-1} \text{s}^{-1} \). From dimensional analysis, we expect that the viscous drag law should have the form:

\[
F_{\text{drag}} \propto \eta u L
\]

In the specific case of a sphere, the force law is known as Stokes' Law:

\[
F_{\text{drag}} = 6\pi \eta R v
\]

\( R \) = radius

- Pressure drag depends on the density of the fluid, \( \rho \), which has SI units of \( \text{kg m}^{-3} \). From dimensional analysis, we expect that the force law for pressure drag should have the form:

\[
F_{\text{drag}} \propto \rho u^2 L^2
\]

It is conventional to write the force law for pressure drag as:

\[
F_{\text{drag}} = \frac{1}{2} C_d \rho A u^2
\]

\( C_d \) = drag coefficient

\( A \) = cross-sectional area

- In general, objects will experience both kinds of drag force. In almost all cases, though, one will predominate over the other. How will you know which one to use?
Activity 3: Terminal Velocity

- A skydiver in free fall will eventually reach a terminal velocity, at which point his velocity is constant. Use Newtonian dynamics to estimate the terminal velocity of a skydiver. Assume a drag coefficient $C_d = 0.3$. The density of air is roughly $1 \text{ kg/m}^3$; you should know or estimate all other relevant quantities. (Use the FANCLAN checklist...)

\[ F_{\text{drag}} \]
\[ F_{\text{y}} \]

\[ \begin{align*}
F_y: & \quad \Sigma F_y = m a_y \Rightarrow F_{\text{drag}} - F_y = m a_y \\
C & = (0, 0) \\
F_y & = mg \\
F_{\text{drag}} & = \frac{1}{2} C_d \rho A v^2 \\
\frac{1}{2} C_d \rho A v^2 - mg & = 0
\end{align*} \]

\[ \begin{align*}
V & = \sqrt{\frac{2mg}{C_d \rho A}} \\
C & = 0.5 \\
\rho & = 1 \text{ kg/m}^3 \\
v & \approx 67 \text{ m/s} \quad 150 \text{ mph}
\end{align*} \]

- Video: Peregrine falcon:
  [http://education.nationalgeographic.com/education/media/terminal-velocity/?ar_a=1](http://education.nationalgeographic.com/education/media/terminal-velocity/?ar_a=1)

- Bonus! The skydiver opens his parachute so that his cross-sectional area increases instantly by a factor of 5. What is the net force on the skydiver at the moment that the parachute opens?

Initially at terminal velocity $F_{\text{drag}} = mg$

but if $A \rightarrow 5A$, then $F_{\text{drag}} \rightarrow 5mg$

so $1mg$ down and $5mg$ up $\Rightarrow 4mg$ up
Activity 5 (from L3a): Putting it together

- A checklist to help you remember all the steps required for Newtonian dynamics problems: FANCLAN
  - F - Free body diagram for all Forces acting on each object
  - A - Axes (choose so they simplify the problem—is the motion in one dimension?)
  - N - Newton's Second Law, in component form, for each object
  - C - Constraints on the motion (equations for a)
  - L - Laws for the forces (equations for various forces $F$)
  - A - Algebra: solve for unknowns; Anything else needed?
  - N - Nonsense? Reflect on answer: does it make sense? Units consistent? Limiting cases?

- I'm going to swing a bucket of water over my head. What is the minimum speed of the bucket that will ensure that the water stays in the bucket? My arm is about 1 meter long; treat the water as the object of interest and think about the conditions required for it to stay on its circular trajectory when it is directly over my head.

\[ y: \sum F_y = ma_y \Rightarrow -F_g - F_N = may \]

\[ a = \frac{v^2}{R} \]

\[ F_g = mg \]

\[ -mg - F_N = m \left( -\frac{v^2}{R} \right) \Rightarrow mg = -F_N \frac{v^2}{R} \Rightarrow g = \frac{v^2}{R} \Rightarrow v = \sqrt{gR} \approx 3 \text{ m/s} \]

\[ \text{glad so well good!} \]