Pre-Reading for Lecture 2b:
Kinematics with Constant Acceleration

• In many instances, we can assume the net force on an object is constant. For instance, a ball falling through the air experiences a constant downward force from gravity, and if we ignore air drag, the net force on the ball will be constant.

• If the average net force is constant, we can derive three kinematic equations that relate the position, velocity, and acceleration of the object at any time. For instance, for motion in one dimension (the y-dimension) we have the following equations:

  Velocity vs. time: \( v_y = v_{0y} + a_y t \)
  Position vs. time: \( y = y_0 + v_{0y} t + \frac{1}{2} a_y t^2 \)
  Velocity vs. position: \( v_y^2 - v_{0y}^2 = 2a_y (y - y_0) \)

In these equations, the y-acceleration \( a_y \) is constant. The initial time is \( t = 0 \) and the final time is just \( t \). When \( t = 0 \), the initial y-position is \( y_0 \) and the initial y-velocity is \( v_{0y} \).

• In Cartesian coordinates, motion in the x-direction is independent of motion in the y-direction, so the kinematic equations for the x-coordinate are basically identical (just replace “y” with “x” in each of the equations above). We can combine the components of first two kinematic equations above into vector kinematic equations:

  Velocity vs. time: \( \vec{v} = \vec{v}_0 + \vec{a} t \)
  Position vs. time: \( \vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2} \vec{a} t^2 \)

(Expressing the third kinematic equation in vector form requires the dot product, which we will introduce later in the course...so don’t worry about it for now.)

• Learning objectives: After this lecture, you will be able to...

  1. Derive and explain Newton’s Third Law.
  2. Explain and derive the kinematic equations for constant acceleration.
  3. Use the kinematic equations to solve 1-D problems involving position, velocity, and time.
  4. Draw graphs showing the relationship between position, velocity, and acceleration for cases in which the acceleration is piecewise constant.
  5. Describe the motion of objects in free fall (projectile motion) using the 2-D equations for kinematics with constant acceleration.
  6. Use the kinematic equations to solve 2-D problems involving position, velocity, and time.
Newton’s 3rd Law (Pre-Video)

- Consider a system of two objects that are isolated from their surroundings. The objects interact through a force that is internal to the system:

- What (if anything) can we say about the total momentum of the system?

- We can define the system any way we like! Take object 1 alone. Is it isolated? What is the nature of its interaction with object 2? What can we say about the momentum of object 1?

- Or, we can take object 2 alone as the system:

- Putting it all together, we find:
Position, Velocity, and Acceleration for a Constant Net Force (Pre-Video)

In many instances, we can assume the net force on an object is constant. For instance, a ball falling through the air experiences a constant downward force from gravity, and if we ignore air drag, the net force on the ball will be constant.

If the net force is constant, and the mass of an object is constant, what must be true about the acceleration of the object? (You’ll need an equation to have a convincing answer...)

\[ F = ma \]
\[ a = \frac{F}{m} \]

If the average net force is constant, we can derive four kinematic equations that relate the position, velocity, and acceleration of the object at any time. Let’s consider motion in one dimension—the y-dimension—as would be the case for an object falling in a straight vertical line under the influence of gravity. First we need to define some quantities:

- At the initial time \( t = 0 \), the object’s position is \( y_0 \), and the object’s y-velocity is \( v_{0y} \).
- The y-component of the acceleration \( a_y \) is constant.

Then, for any later time, the following four relationships will hold (as long as the net force remains constant):

1. **Velocity vs. time:** \[ v_y = v_{0y} + a_y t \]
2. **Average velocity:** \[ \langle v_y \rangle = \frac{v_i + v_f}{2} = \frac{y - y_0}{t} \]
3. **Position vs. time:** \[ y = y_0 + v_{0y} t + \frac{1}{2} a_y t^2 \]
4. **Velocity vs. position:** \[ v_y^2 - v_{0y}^2 = 2a_y (y - y_0) \]

Solve equation 1 (velocity vs. time) for the acceleration \( a_y \).

Show that equation 1 is merely the definition of constant acceleration: \[ a_y = \frac{\Delta v_y}{\Delta t} \]

(Hint: The initial time is \( t_i = 0 \); the final time is \( t_f = t \). What is the initial velocity? The final velocity? What does “delta” mean?)

Rearrange 1.

\[ a_y = \frac{v_y - v_{0y}}{t} \]
\[ v_f = v_i + \frac{v_y - v_{0y}}{t_f - t_i} \]
\[ \Delta v_y = \frac{v_f - v_i}{t_f - t_i} \]
Physical Sciences 2: Lecture 2b

**Activity 1: Kinematics in 1-D**

- Consider motion in *one dimension*: for instance, an object falling in a straight vertical line under the influence of gravity. The magnitude of the gravitational acceleration is a constant $g = 9.81$ m/s$^2$. The $y$-components of the kinematic equations become:

  **Velocity vs. time:**
  \[ v_y = v_{0y} + a_y t \]

  **Position vs. time:**
  \[ y = y_0 + v_{0y} t + \frac{1}{2} a_y t^2 \]

  **Velocity vs. position:**
  \[ v_y^2 - v_{0y}^2 = 2a_y (y - y_0) \]
  (*this equation can be derived from the other two.*)

1. You fall, starting from rest, from a second-story window and land on the ground. Just before you hit the ground, you are traveling at a speed of 10 m/s. How long were you falling?

2. You bend your legs as you strike the ground. During the impact, your center of mass travels a distance of 0.5 meters until it comes to rest. Assuming constant acceleration during the impact, find the magnitude of the acceleration and the net force on your body as you land. (Assume your mass is 75 kg.)

- **Bonus!** How would the net force change if you land with *stiff* legs, so your center of mass travels 1 mm during the impact?
Activity 2: Graphical representations

- Sample: Using the definitions of \( y \), \( v_y \), and \( a_y \), sketch graphs of \( a_y \) and \( y \) given \( v_y \) below.

1. Now it’s your turn—fill in the missing graphs.

2. Stump your friends! You draw a graph of \( v_y \) and have your friends fill in the others.
Two-dimensional kinematics and projectile motion

- One great virtue of Cartesian coordinates is that the motion in the $x$ and $y$ directions are entirely independent of one another.

So we can write, for instance:

**$x$-component:** \[ x = x_0 + v_{0x}t + \frac{1}{2}a_xt^2 \]

**$y$-component:** \[ y = y_0 + v_{0y}t + \frac{1}{2}a_yt^2 \]

- A very important special case is **free fall**, when an object is only under the influence of gravity. If we choose conventional coordinate axes, what are the components of the acceleration for an object in free fall?

- Cool example of projectile motion (free fall):

Hippy jump
Activity 3: Kinematics in 2-D

1. You are serving a tennis ball by hitting it horizontally at a height of $h_B = 2.50 \text{ m}$ with an initial speed $v_0$. The ball must clear the net at a height $h_N = 0.90 \text{ m}$ and a distance of $D = 15.0 \text{ m}$. Calculate the minimum speed $v_0$ required for the ball to clear the net.

2. **Bonus!** You fire a projectile at an initial speed $v_0 = 50 \text{ m/s}$ at an angle $\theta = 60^\circ$ above the horizontal. The projectile lands on a slope of angle $\phi = 40^\circ$ below the horizontal. Find the distance $d$ to the spot where it lands.
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One-Minute Paper

Your name: _____________________________       TF: _____________________________

Names of your group members:  _________________________________
_________________________________
_________________________________

• Please tell us any questions that came up for you today during lecture. Write “nothing” if no questions(s) came up for you during class.

• What single topic left you most confused after today’s class?

• Any other comments or reflections on today’s class?