PART A (Graded by Sathvik)

PROBLEM 1
(4 points, suggested length of 1/3 page)

Determine whether the series $1 + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \ldots$ converges or diverges. If it converges, calculate the sum of the series. If it diverges, prove why. (Hint: consider the harmonic series!)

Solution.

PROBLEM 2
(3+3 points, suggested length of 1/2 page)

A ternary sequence is like a binary sequence, only with 0, 1 and 2 as possible digits instead
of solely 0 and 1. Find recurrence relations and initial conditions that describe the number of $n$-digit ternary sequences with:

(A) no occurrence of the subsequence “210”.
(B) an even number of 0s.

Solution.

PART B (Graded by Theresa)

PROBLEM 3

(2+2 points, suggested length of 1/4 page)

Evariste Galois (a great mathematician who actually died in a duel at age 19) is fighting a duel to the death with Gaston. They fire alternately, with Evariste going first: EGE... Evariste is a terrible shot, and has only one chance in five of killing Gaston with any given shot. Gaston is a somewhat better marksman, and has one chance in four of killing Evariste
with any given shot. Either a shot kills the opponent, or it misses completely.

(A) By summing a geometric series, determine the probability that Evariste wins the duel.

*Hint:* Evariste wins if both duelists miss $r$ times, and he then hits.

(B) To check your answer, sum a series to calculate directly the probability that Gaston wins.

Solution.

PROBLEM 4

(3 + 3 points, suggested length of 1/3 page each)

Find functions fitting the following recurrences, and prove your answers by induction.

(A) $f(1) = 1, f(n) = f(n/2) + n/2$

(B) $g(0) = 0, g(n) = g(n - 1) + 2n + 1$
Solution.