Harvard University
Computer Science 20

Problem Set 8
Due Tuesday, April 8, 2018 at 11:59pm.

PART A (Graded by Ahmed)

PROBLEM 1 (5 points)

Fill in the following table by checking off the boxes for each relation R iff \( f R g \) for the functions \( f \) and \( g \) on that row. For example, check off the top-left box iff \( 2^x = O(2^x + 2x) \) and the lower-right box iff \( x! \sim x^x \). No justifications are necessary.

<table>
<thead>
<tr>
<th>( f(x) )</th>
<th>( g(x) )</th>
<th>( O )</th>
<th>( o )</th>
<th>( \Omega )</th>
<th>( \Theta )</th>
<th>( \sim )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 2^x )</td>
<td>( 2^x + 2x )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( (\frac{1}{7})^x )</td>
<td>500</td>
<td></td>
<td></td>
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<tr>
<td>( 10^x )</td>
<td>( 11^x )</td>
<td></td>
<td></td>
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<td></td>
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<tr>
<td>( x + \frac{1}{x} )</td>
<td>( \log(2^x) )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( x! )</td>
<td>( e^x )</td>
<td></td>
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<td></td>
<td></td>
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</tr>
<tr>
<td>( x! )</td>
<td>( e x^x )</td>
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</tbody>
</table>

Solution.
PROBLEM 2 (3 points, suggested length of 2/3 page)

Prove that $\Theta$ is an equivalence relation. That is, prove that $\Theta$ is symmetric, transitive and reflexive.

Solution.
PROBLEM 3 (1+1+1 points, suggested length of 1/3 page)

The last event that your campus group promoting civil discourse across party lines (Problem Set 1) was such a success that you’ve decided to hold another. Once again, you decide to invite 5 Republicans, 5 Democrats and 2 independents and assign them seats at a round 12-person table. The direction the guests are facing does not matter, so you do not distinguish between seating assignments that differ only by rotation.

(A) If seats are allocated by party, where the independents are treated as one party, how many unique seating assignments are there?

(B) At the last event, the Democrats didn’t communicate much with the other parties. How many unique seating assignments are there where not all the Democrats are seated together?

(C) The president of the campus group wants to instead assign seats by individual so that members of the same party are not considered interchangeable. She also wants to consider seating assignments that are reflections of each other to be indistinguishable, since we only care who is seated next to whom. How many unique seating assignments are there following this plan?

Solution.
PART B (Graded by Allison)

PROBLEM 4 (1+1+2 points)

Recall that simple undirected graphs are graphs that contain at most one edge between any pair of vertices and no self-loops. Assume all graphs are simple and undirected in this question. For each of the following, provide a short (three sentences or less) explanation for your answer.

(A) How many edges are there in the complete graph of \( n \) vertices?
(B) A complete bipartite graph is a bipartite graph where every vertex in one partition is connected every vertex in the other partition. How many edges are there in the complete bipartite graph with \( n \) vertices in each partition?
(C) How many isomorphisms are there from a complete graph on \( n \) vertices to itself?

Solution.
(A) A straight is a 5-card poker hand in which all five cards are in sequence by rank e.g. a 7, 8, 9, 10 and Jack. How many ways are there to be dealt a straight, if the order in which the cards are received does not matter? Note: Aces can count as both the low-card (ranked below the 2) and high card (ranked above the King).

(B) A flush is a 5-card poker hand that has in which all five are the same suit. How many ways are there to be dealt a flush?

(C) How many hands are either a straight or a flush?

Solution.