Mathematical modeling

Applied Mathematics/Engineering Science 115 (Spring 2018)

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Date & time: M/W 2:30-4pm
Location: Pierce Hall 301

Bibliography: Material from several textbooks and other sources will be used (see list at end of syllabus); the source materials for all lectures, including Matlab programs used in class, may be found on the course web site.

Web: www.courses.fas.harvard.edu/1768

Notes, readings, and Matlab codes will be posted on the webpage. Always feel free to write us with any questions!

1. Overview

Mathematical models are ubiquitous, providing a quantitative framework for understanding, prediction, and decision-making in nearly every aspect of life, ranging from the timing of traffic lights, to the control of the spread of disease, to resource management, to sports. They also play a fundamental role in all natural sciences and increasingly in the social sciences as well. This course provides an introduction to modeling through in-depth discussions of a series of examples, and hands-on exercises and projects that make use of a range of continuous and discrete mathematical tools.

2. Administrative

Prerequisites: Applied Mathematics 21a and 21b, or Mathematics 21a and 21b or permission of instructor. Knowledge of some programming language is helpful, but not necessary, as we’ll be introducing Matlab for those with no previous experience.

Computer skills: Computer labs will be run in Matlab (see note on Python below), and as part of the course, students will therefore gain facility with this programming language. Students should download and install the Matlab system on their computers from the FAS software site (https://downloads.fas.harvard.edu/download). You can read find Matlab tutorials online (e.g. here: http://www.mathworks.com/help/matlab/getting-started-with-matlab.html?s_cid=learn_doc or on the course webpage. Additional introductions to Matlab will be given as needed. If you are more comfortable with Python, feel free to use that for your work. We will gradually make Python versions of
the demonstration codes available as well.

**Lectures and workshops:** In addition to the lectures, the class will have in-class exercises and workshops. Matlab will use in many of the exercises and workshops. Each group (see below) should have at least one laptop with Matlab installed. Python can be used as well. The groups are expected to turn in a short report summarizing findings from the workshop.

**Homework assignments:** We will assign homework on some basic problem solving skills. There will be a total of two to three assignments. They will be due during weeks when there is not a project due.

**Late Homework Policy:** We accept late problem sets up to 4 days after the due date, but will apply a 15% penalty for each 24h after the due date. The penalty is applied only to the parts that are turned in late. This only applies to problem sets. We cannot accept late group projects.

**Group projects:** Group projects will be completed by 3-person teams (to be formed over the first two weeks). These projects will further explore the models discussed in class. Groups must consult with their assigned TF during office hours to develop reasonable projects, and will present their findings in class. A brief written report (5-10 pages) is expected from each group for each of the projects. *Contributions from each group member should be clearly noted in the report.*

**Individual final project:** During the last weeks of the semester, each individual will carry out a final project investigating a new model or carrying out a significant extension of a model discussed in class. Student creativity in selecting and refining the project topic will be important in evaluation of the project. During this time period, homework will not be assigned.

**Late Final Project Policy:** We accept late final projects up to 4 days after the due date, but will apply a 15% penalty for each 24h after the due date.

**Grading:** In-class workshop and exercises 15%; Homework assignments 10%; Group projects 30%; Judging other groups’ presentations 5%; Individual final project 40%. To encourage collaboration, grading will not follow a given distribution and instead will be based on the level of excellence.

**Policy on Collaboration:** Collaboration on group projects is of course permitted. The use of sources beyond your team and the course materials posted on this year's course web site should be explicitly referenced in your presentations and written work. The final project is intended to be an individual piece of work and all collaboration must be explicitly referenced. All work on it should be entirely your own and you must use appropriate citation practices to acknowledge the use of books, articles, websites, lectures, discussions with others, including the course staff and your teammates, etc., that you have consulted to complete your project. The Harvard Guide to Using Sources is available at [http://usingsources.fas.harvard.edu](http://usingsources.fas.harvard.edu).
Accommodations for students with disabilities
Students with accessibility issues should get in touch with the Accessibility Services office at Accessibility@dcemail.harvard.edu or 617-998-9640.

3. Topics (tentative)

The topics to be covered are listed below, which will be fine-tuned during the course based on students’ interest. You may find more information in the overview slides on the course website (under Modules).

3.1: Introduction; Modeling philosophy

Why model? What’s a good model? (As simple as possible, equations well motivated by data or reasoning, aids our understanding, and able to surprise us and provide new insight/prediction not obvious beforehand). Model validation. Simple and complex models, simulation vs. modeling, stochastic vs. deterministic.

3.2: Population dynamics, single species

We will start simple by considering a single species with limited resources.

Deterministic approach: logistic equation, geometric approach, linearized stability analysis, numerical solutions of ordinary differential equations, sensitivity to parameters, bifurcation, hysteresis.

Stochastic approach: equations for the probability distribution, simulating stochastic processes with Monte Carlo.

In-class exercises: IEEE floating point numbers; balancing a stick; insect outbreaks; snowball Earth; discuss the pros and cons of the deterministic and stochastic approaches.

3.3: Population dynamics, competition of species

Two competing species: deadly survival struggle between sheep and Rabbits. Intro to phase plane, fixed points, stability, classification of linear systems, nonlinear limit cycles. Predator-prey oscillations, Lotka-Volterra equations. Application to epidemiology, marriage and divorce, and love affairs...yup...love affairs.

In-class exercises: Examine how a love affair is affected by the individuals’ personalities; Graphic analyses of the behavior of a model of competition

Workshop: Modeling an epidemic with deterministic, probability distribution, and individual agent approaches

3.4: Probabilistic models

Monte Carlo (Buffon’s needle, profit vs. risk, Bernoulli trials, Poisson distributions) Markov Chain (fundamental matrix, steps to absorption, mean first passage time),
Applications to the inventory problem, the queuing problem, genetics, gambling, and the Internet and Google’s PageRank algorithm.

**In-class exercises:** Gambler’s ruin with Monte Carlo and Markov Chain

**Workshops:** Queuing models and Genetics

### 3.5: Traffic

Models for car following, delayed differential equations, stability analysis. Simple continuum theories, method of characteristics, waves and shocks, other applications such as secondary oil recovery.

**In-class exercises:** Traffic congestion with car following and with continuum theory.

### 3.6: Diffusion


**Workshop:** Lord Kelvin and the age of Earth (analytical and numerical solutions)

### 3.7: Optimization models

Constrained optimization; Lagrange multipliers; shadow price; linear programming, integer programming

**In-class exercises:** laptop manufacturing; nutritious diet on a budget

### 3.8: Applications of graph theory

Basic graph theory; graph coloring; shortest path; minimum spanning tree; maximum flow; matching; vertex cover

**In-class exercises:** scheduling final exams; summer rentals; power line for villages; policing a city

### 3.9: Data fitting and model selection

Least squares, statistical interpretation, uncertainty estimates.

**In-class exercises:** (re-)discover Kepler’s 3rd law

### 3.10: Statistical Models (supervised and unsupervised learning)

How to build a model from big data? Regression; Classification; tradeoff between model complexity, bias, and variance; Cluster analysis, Principal Component Analysis, Neural network.
In-class exercises: Supreme Court; wine tasting

Workshop: Bio-motion: what can we learn about an individual by how she/he walks? (Principal Component Analysis for dimensional reduction and for pattern recognition)

4. References


