f) [5 pts] A sparrow flying through air and a goldfish swimming in a pond have very
dissimilar means of biomechanical self-propulsion: the bird has wings that flap up and
down, and the fish has fins that swish back and forth. This dissimilarity is due primarily
to a difference in which of the following properties of the fluid flow surrounding the
animals?

A. viscosity  
B. density  
C. Reynolds number  
D. surface tension

a) [5 pts] Two identical tanks are filled to the same height with liquid. One contains
water, and the other contains mercury, with a density 13 times that of water. A 1-cm
diameter hole is punched simultaneously near the bottom of each container so that liquid
begins to leak out. Which tank will empty out first?

A. the water  
B. the mercury  
C. both will empty at the same time

b) A solid sphere sinks in a fluid (at low Reynolds number) with a certain terminal velocity. A
second sphere, made of the same material but with twice the radius of the first sphere, is placed in
the same fluid. Compared with the first sphere, the larger sphere will reach a terminal velocity
that is:

A. half as fast  
B. the same  
C. \( \sqrt{2} \) times as fast  
D. twice as fast  
E. four times as fast

c) When a sphere of radius \( R \) is dragged at a speed \( v \) through liquid with viscosity \( \eta \), the force \( F \)
required to do the dragging is \( F = 6\pi \eta R v \) at low Reynolds number. Similarly, if the sphere is
rotated with an angular velocity \( \omega \) in the same viscous fluid, a certain torque \( \tau \) is required to
produce the rotation. Which formula below is dimensionally correct?

A. \( \tau = 8\pi \eta \omega R^3 \)  
A. \( \tau = 8\pi \eta \omega R^2 \)  
A. \( \tau = 8\pi \eta \omega^2 R^2 \)  
A. \( \tau = 8\pi \eta \omega R^4 \)
d) Suppose that both a feather and an elephant (!) fall from a tall tree. Which encounters the greatest force of air resistance while falling to the ground?
   A. the elephant
   B. the feather
   C. both encounter the same force of air resistance

e) A U-tube (both ends are open) contains a quantity of liquid that does not mix with water. A small amount of water (density \( \rho = 1.0 \times 10^3 \text{ kg/m}^3 \)) is poured into the left end of the tube. The system comes to equilibrium in the configuration shown below, with \( h_1 = 15 \text{ cm} \), \( h_2 = 12 \text{ cm} \), and \( h_3 = 17 \text{ cm} \). What is the density of the liquid?

![U-tube diagram]

   A. \( 0.6 \times 10^3 \text{ kg/m}^3 \)
   B. \( 0.8 \times 10^3 \text{ kg/m}^3 \)
   C. \( 1.3 \times 10^3 \text{ kg/m}^3 \)
   D. \( 1.7 \times 10^3 \text{ kg/m}^3 \)
   E. \( 2.5 \times 10^3 \text{ kg/m}^3 \)

Problem 1 (continued)

e) [5 pts] Water is flowing from left to right in a large pipe. At a certain point, the pipe narrows slightly. Consider a small speck of dirt carried along by the flow. Which statement below correctly explains what happens to the speck as it moves from point A to point B as shown?

   A. It slows down, because viscous drag increases in the constricted region.
   B. It slows down, because pressure drag opposes its motion.
   C. It speeds up, because the pressure is higher on the left than the right.
   D. It speeds up, because the Reynolds number is higher at point B than at point A.
   E. It continues to move at the same speed, because the flow rate does not change.
g) [5 pts] Blood flows through a large artery with radius $R_1$. This artery divides into two smaller arteries, each with radius $R_2 = R_1 / 4$, as shown below. At this type of junction turbulent blood flow can sometimes be heard as a “murmur” on a stethoscope. What is the relationship between the Reynolds number in the main artery (Re$_1$) and the Reynolds number in the smaller arteries (Re$_2$)?

A) $\text{Re}_2 = \frac{1}{2} \text{Re}_1$
B) $\text{Re}_2 = \text{Re}_1$
C) $\text{Re}_2 = 2 \text{Re}_1$
D) $\text{Re}_2 = 4 \text{Re}_1$

![Diagram of blood flow through a large artery with two smaller arteries, each with radius $R_2 = R_1 / 4$, showing the Reynolds numbers.]

b) [5 pts] Blood pressures are normally measured with the upper arm at the same level as the heart. If you hold your arm up over your head instead, how will this affect the measurement of your blood pressure?

(a) It will be the same as normal   (b) It will be higher than normal
(c) It will be lower than normal   (d) It will fluctuate above and below normal
Problem 2: Microcapillary [20 pts]

Consider a small droplet of water of radius $R = 100 \, \mu m$ suspended in air. A microcapillary tube of inner radius $r = 5 \, \mu m$ and length $L = 50 \, \mu m$ is stuck into the droplet, and the water begins to drain out the tube. Ignore the effects of gravity. Water has viscosity $\eta = 10^{-3} \, Pa \cdot s$ and surface tension $\gamma = 0.072 \, N/m$.

a) [15 pts] Calculate the instantaneous flow rate $Q$ of the water as it starts to drain from the end of the tube.

b) [5 pts] As water drains through the tube, the droplet will shrink. Will the flow rate in the tube increase, decrease, or stay the same? Briefly justify your answer.
2. In class you saw a demonstration of Poiseuille’s Law in which water flowed out of an open tank through a long narrow tube. As shown in the diagram, the water level in the tank is \( h = 10 \text{ cm} \) above the opening at the bottom of the tank, the cylindrical tube has a length of \( L = 1 \text{ m} \) and an interior radius of \( R = 1 \text{ mm} \), and the tube is a height of \( H = 1 \text{ m} \) above the ground:

![Diagram of water flow](image)

a) What is the flow rate \( Q \) in the cylindrical tube? (Water has a density \( \rho = 10^3 \text{ kg} \cdot \text{m}^{-3} \) and a viscosity \( \eta = 10^{-3} \text{ kg} \cdot \text{m}^{-1} \cdot \text{s}^{-1} \).)

b) What is the horizontal distance \( d \) that the water travels before it strikes the ground?

c) If you replaced the water with a *more viscous* fluid, the distance \( d \) would (circle one):

- increase
- remain the same
- decrease
6. There has been tremendous recent interest in using wind as a source of power. The turbine shown at right has blades that are about $R = 40$ meters long.

   a) Estimate the Reynolds number for air flow at a speed of 10 m/s around this wind turbine. Air has a viscosity of $\eta = 1.8 \times 10^{-5}$ kg·m$^{-1}$·s$^{-1}$ and a density of $\rho = 1.3$ kg·m$^{-3}$.

   b) Wind turbines require strong winds in order to produce substantial amounts of power. Using dimensional reasoning, determine an expression for the power produced by a wind turbine in terms of the wind speed $v$, the radius $R$ of the turbine, and any relevant physical properties of air given in part (a) above. Please show your reasoning clearly. (Hint: your answer to part (a) should help you choose which physical properties are relevant to this system.)
Problem 3: Air Pipe [15 pts]

Air flows from left to right through the cylindrical pipe shown. The radius of the pipe is \( R = 10 \) cm, but there is a constricted region in which the radius of the pipe drops to \( R/2 \). The side tube contains water. The height difference of the water level between the two ends of the tube is \( \Delta h = 4 \) cm, as shown.

Calculate the speed \( v_1 \) of the air as it enters the tube. You may use the following information: \( \rho_{\text{air}} = 1.2 \) kg/m\(^3\), \( \rho_{\text{water}} = 1000 \) kg/m\(^3\), \( P_{\text{atm}} = 100 \) kPa.
A viscous fluid ($\eta = 10^{-1} \text{ kg} \cdot \text{m}^{-1} \cdot \text{s}^{-1}$) flows from left to right through the system illustrated below. (This fluid flow is characterized by a low Reynolds number.) At the left, the fluid is introduced to a tube with inner radius $R_1 = 2 \text{ mm}$ at an absolute pressure of $P_0 = 2 \text{ atm } (2 \times 10^5 \text{ Pa})$. After a distance of 1 m, this tube splits into two identical tubes, each with an inner radius of $R_2 = 1 \text{ mm}$ and a length of 1 m. At the right, the fluid flows out of the tubes into the atmosphere ($P_2 = 1 \times 10^5 \text{ Pa}$). What is the total flow rate $Q$ through this system? (Hint: as part of your answer, you will need to find the pressure $P_1$ at the junction where the tubes split.) You may ignore any effects of gravity.

\[ Q = \]