Physical Sciences 2

Exam 1 (Group)
Thursday, September 29, 2016

Student name: Answer illegible Section TF: __________________________

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Do not turn the page until you are told to begin. You will be given 90 minutes to complete this exam. Show all your work on the exam itself; no credit will be given for anything written on other paper. Please box your final answer to each calculation.

You may use a calculator if you have brought one. You may refer to one 8.5”x11” sheet of notes (both sides), which must be in your own handwriting. Turn in your notes along with the exam when time is called.

This exam contains 6 sheets of paper (including this one), with 6 problems.

Do not write in the following table; it will be used for grading.

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Problem 1: Multiple Choice / Short Answer [20 points]

For each of the multiple choice questions, circle the letter(s) corresponding to the best answer(s) from the options given. Each question is worth 5 points; partial credit will be given for those problems asking you to “Circle all that apply.”

a) An isolated 10 kg space probe floats in outer space at a speed of 5 m/s towards the right. It spontaneously bursts into 4 pieces that fly off in different directions at great speeds. *Circle ALL that apply*...

   A) After the explosion, the sum of momentum for all 4 pieces adds up to zero.
   B) After the explosion, the center of mass of the system must be at rest.
   C) After the explosion, the sum of momentum for all 4 pieces adds up to a magnitude of 50 kg·m·s⁻¹.
   D) Momentum cannot be conserved for this system.
   E) After the explosion, the center of mass of the entire system must be moving at 5 m/s to the right.

b) A 2 kg pigeon and a 4 kg hawk are flying horizontally toward each other. The moment after they collide, they are both precisely at rest. *Circle ALL that apply*...

   A) The hawk and the pigeon were initially flying with the same velocity.
   B) The hawk and the pigeon were initially flying with the same speed (but different velocities).
   C) The pigeon’s initial speed was greater than the initial speed of the hawk.
   D) Before the collision their combined center of mass was twice as far from the pigeon as it was from the hawk.
   E) Before the collision, the momentum vector of the hawk plus the momentum vector of the pigeon was equal to zero.

*continued on next page*...
c) Two spheres A and B of equal mass are located at some distance from each other. Sphere A is at rest while sphere B moves away from A at speed $v$. What must be true about the center of mass of the two-sphere system? *Circle ALL that apply...*

A) It remains at rest.
B) It moves with speed $v$ away from A.
C) It moves with speed $v$ toward A.
D) It moves with speed $\frac{1}{2}v$ away from A.
E) It moves with speed $\frac{1}{2}v$ toward A.

d) [*Short answer*] When a force $F$ is applied tangentially to a sphere of radius $R$, this force exerts a torque $\tau$ (Greek letter tau) that can cause the sphere to rotate. In this case, the torque is $\tau = RF$.

You have a sphere that is immersed in a fluid of viscosity $\eta$, and you wish to apply a torque $\tau$ so that the sphere makes one full revolution every $T$ seconds. Find a proportionality that expresses how the torque $\tau$ could depend on the sphere radius $R$, the viscosity $\eta$, and the time $T$. (Recall that viscosity has SI units of $\text{kg} \cdot \text{m}^{-1} \cdot \text{s}^{-1}$.)

Your answer should be in the form of a proportionality: $\tau \propto \ldots$

$$\tau = RF = (L) \left( \frac{ML}{T^2} \right) = \frac{ML^2}{T^2}$$

So $\tau = (R)^3 \cdot (\eta)^2 \cdot (T)^{-1}$

$$\frac{ML^2}{T^2} = (L)^3 \left( \frac{M}{LT} \right)^1 \cdot (T)^{-1} \Rightarrow \text{then to balance} \ T$$

get $\tau \propto \frac{R^3 \eta}{T}$
Problem 2: Launch it! [15 points]

Consider the apparatus shown at right. A spring-loaded launcher of mass $m_l = 100 \text{ g}$ is free to slide without friction on a horizontal air track. Initially the launcher is at rest with a steel ball of unknown mass $m_B$ inside. When the spring is released, the ball is projected horizontally at a speed of 5 m/s relative to the floor; the launcher recoils with a speed of 1 m/s relative to the floor.

a) [5 pts] Calculate the mass of the ball, $m_B$. (The launcher mass $m_l$ does not include the mass of the ball.)

$$\vec{P}_i = \vec{P}_f$$

$$\text{Conserv.: } m_l V_{lx} + m_B V_{bx} = m_l V_{lf} + m_B V_{bf}$$

$$= 0 = 0$$

$$0 = m_l V_{lf} + m_B V_{bf}$$

Solve $$m_B = m_l \left( \frac{-V_{lf}}{V_{bf}} \right)$$

$$= 100 \text{ g} \left( \frac{5}{-(-2.5)} \right) = 20 \text{ g}$$

b) [10 pts] As noted in the diagram, the ball is launched from a height $y_0 = 1 \text{ m}$ above the floor. Calculate the angle $\theta$ at which the ball strikes the floor; you may neglect air resistance.

Need components of velocity

Free fall: $V_x = V_{0x} = 5 \text{ m/s}$

$V_y = V_{0y} - gt$

$V_y = -gt$

Solve for $t$: $y = y_0 + V_{0y}t - \frac{1}{2} gt^2$

$0 = (1 \text{ m}) + 0 - \frac{1}{2} (9.8 \text{ m/s}^2) t^2$

$gt = \sqrt{\frac{2y_0}{g}}$ so $V_y = -\sqrt{2y_0g}$

$= 4.4 \text{ m/s}$

$\tan \theta = \frac{4.4}{5}$ so $\theta = \arctan(0.88) = 41^\circ$
Problem 3: Center of Mass [15 points]

The diagram to the left shows a collection of uniform masses distributed on an x-y plane.

The system consists of two uniform squares each of mass $m$, and one uniform circular hoop of mass $2m$ placed with one end on the origin of the reference axis as shown.

The diagram is drawn to scale using 1 cm graph paper.

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a) [6 pts] Calculate the position vector for the center of mass (CM) of this system. Please give your answer in component vector form. (Hint: make good use of symmetry).

1. $\vec{r}_{1cm} = (-5, 4) \text{ cm}$
2. $\vec{r}_{2cm} = (5, 4) \text{ cm}$
3. $\vec{r}_{3cm} = (0, 2) \text{ cm}$

$$\vec{r}_{CM, \text{ total}} = \frac{1}{M_{\text{tot}}} (m_{1} \vec{r}_{1cm} + m_{2} \vec{r}_{2cm} + m_{3} \vec{r}_{3cm})$$

$$= \frac{1}{4m} (1m(-5, 4) \text{ cm} + 1m(5, 4) \text{ cm} + 2m(0, 2) \text{ cm})$$

$$= \frac{1}{4} (-5 + 5 + 0, 4 + 4 + 4) \text{ cm} = \frac{1}{4} (0, 12) \text{ cm}$$

$$= (0, 3) \text{ cm}$$

continued on next page...
b) [9 pts] A fourth object that has a mass of 4m is added to the system. Where would the fourth mass need to be positioned in order to place the total CM for the entire system precisely at the origin? Please give your answer in component vector form and show all of your work.

\[
\vec{r}_{cm, new} = 0 = \frac{1}{m_{tot, new}} \left( 4m \ (0, 3)_{cm} + 4m \ (x, y) \right)
\]

\[
\Rightarrow (0, 0)_{cm} = \frac{1}{8m} \left( x, 12m \cdot cm + 4m \cdot y \right)
\]

\[
\Rightarrow x = 0 \quad y = -3cm
\]
Problem 4: No Gravity [20 points]

Two astronauts are out in space, Sandra of mass $m_1$ and George of mass $m_2$. They are floating above the spaceship's platform. George is moving at speed $v_2$ parallel to the platform, so he's having trouble making it back to the ship. Sandra helps him get to the platform. She floats toward where he is going to be. She moves at a speed $v_1$ and angle $\theta$ from the horizontal (see figure) and when they meet, she holds on to him. Clutching each other, they move at a constant velocity until they land on the platform a vertical height $h$ below the location of where Sandra grabs George.

a) [10 pts] Find an expression for the components of the final velocity of Sandra and George together after they have grabbed one another. Your answer could depend on $m_1$, $m_2$, $v_1$, $v_2$, $h$, $\theta$, and any relevant physical constants. (Assume that all of these quantities are positive.)

Conservation of momentum

$\mathbf{\dot{P}}_{i,\text{tot}} = \mathbf{\dot{P}}_{f,\text{tot}}$

Sandra

$\mathbf{\dot{P}}_{i,S} = (m_1 v_1 \cos \theta, -m_1 v_1 \sin \theta)$

George

$\mathbf{\dot{P}}_{i,G} = (m_2 v_2, 0)$

$\mathbf{\dot{P}}_{i,\text{tot}} = \mathbf{\dot{P}}_{i,S} + \mathbf{\dot{P}}_{i,G} = \mathbf{\dot{P}}_{f,\text{tot}}$

$\Rightarrow (m_1 v_1 \cos \theta, -m_1 v_1 \sin \theta) + (m_2 v_2, 0) = (m_1 + m_2) \mathbf{\dot{V}}_{f,\text{tot}}$

$\Rightarrow (m_1 v_1 \cos \theta + m_2 v_2, -m_1 v_1 \sin \theta) = (m_1 + m_2) \mathbf{\dot{V}}_{f,\text{tot}}$

$\Rightarrow \mathbf{\dot{V}}_{f,\text{tot}} = \frac{1}{m_1 + m_2} (m_1 v_1 \cos \theta + m_2 v_2, -m_1 v_1 \sin \theta)$

continued on next page...
b) [6 pts] Find an expression for the time $T$ that elapses from when Sandra grabs George until they land on the platform together. Your answer could depend on $m_1, m_2, v_1, v_2, h, \theta,$ and any relevant physical constants.

$$t = \frac{\Delta x}{v} \quad \text{in } y\text{-direction} \quad \text{this is} \quad t = -\frac{h}{v \sin \theta}$$

$$\Rightarrow t = \frac{\frac{-h}{m_1 v_1 \sin \theta}}{m_1 + m_2}$$

$$= \frac{h (m_1 + m_2)}{m_1 v_1 \sin \theta}$$

c) [4 pts] Suppose Sandra were not around, and George found himself floating, alone, horizontally past the platform. Explain in one sentence one possible way that he could make himself land on the platform. (Assume that George is far enough from the platform that he can't reach out and grab it . . . )

If he threw something like his helmet or a wrench in the opposite direction ($-v_y$) of the platform.
Problem 5: Up the proverbial ramp
[15 points]

A wooden block of mass $M$ is projected upwards with an initial velocity $v_0$ along an inclined plane. The block starts from point A and moves in a straight line up the inclined plane for a time interval $T$, at the end of which it stops. The coefficient of kinetic friction between the block and the plane is $\mu_k$.

a) [9 pts] Find an expression for the time $T$ that elapses before the block comes to a stop. Your answer could depend on $M$, $v_0$, $\mu_k$, and any relevant physical constants.

b) [6 pts] Find an expression for the distance $D$ that the block travels up the plane. Your answer could depend on $M$, $v_0$, $\mu_k$, and any relevant physical constants.
Problem 6: Swinging a stone [15 pts]

A child ties a stone of mass $m = 0.5 \text{ kg}$ to a nylon string and swings it around in a vertical circle, as shown. The string will break if the tension force exceeds $F_{\text{max}} = 400 \text{ N}$. Assume the child swings the stone at constant speed $v$.

Calculate the maximum value of $v$ such that the string will not break at the bottom of the circle. Show all your work clearly (FANCLAN would be a good idea here).

\[ \mathbf{F}_r \]
\[ \mathbf{F}_{\text{grav}} \]
\[ \mathbf{F}_T = \mathbf{F}_r - \mathbf{F}_{\text{grav}} \]

\[ mg = \frac{V^2}{L} \]
\[ F_T = mg \]

\[ m \frac{V^2}{L} = F_T - mg \]
\[ \text{solve: } F_T = \frac{m V^2}{L} + mg \]

If $F_T = F_{\text{max}}$,

\[ \text{solve for } V^2 = L \left( \frac{F_T}{m} - g \right) \]

or
\[ V = \sqrt{L \left( \frac{F_{\text{max}}}{m} - g \right)} = \sqrt{(1 \text{ m}) \left( \frac{400 \text{ N}}{0.5 \text{ kg}} - 9.8 \text{ m/s}^2 \right)} \]

\[ = 28.1 \text{ m/s} \]

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