Fluids – Basic Principles

Fawwaz Habbal
School of Engineering and Applied Sciences
Harvard University
Interfaces and Surface Tension
Interfaces

- Interface: Geometric Surface that delimits 2 fluids
- Separation depends on molecular interactions and Brownian diffusion
• Interface: Geometric surface that delimits 2 fluids
• Simplified view:

At interface: different energies

Interaction between molecules
• If $U$ is the total cohesive energy per molecule and $d$ is a characteristic molecular dimension, $d^2$ is its surface, then the energy loss (surface tension) is given by:

$$\gamma = \frac{U}{2d^2}$$
Laplace’s Law

- Minimization of surface energy, create curvature of fluids on other surfaces (fluids)
- Curvature $\sim 1/R$
Laplace’s Law

- Minimization of surface energy, create curvature of fluids on other surfaces (fluids)
- Curvature $\sim 1/R$
- Laplace’s Law, the change in pressure is related to the curvature of the surface.

For a sphere:  $\Delta P = 2 (\gamma / R)$
For a cylinder:  $\Delta P = (\gamma / R)$
Surface Tension

- Droplet on a surface
  - Forces on cross section of drop
  - Surface tension along periphery
  - Pressure on section area
  - Pressure difference outside/inside drop

\[ \text{Force} = \Delta PA = \pi r^2 \Delta P \quad \text{Surface Tension} = 2\pi r \gamma \]

\[ \gamma = \frac{r}{2} \frac{\Delta P}{2} \]
Surface Tension

- Cohesive property of the surface of a liquid
- Caused by attractive forces between liquid molecules.
- A.k.a. surface energy
- Defined as the ratio of the surface force $F$ to the length $l$ along which the force acts:

$$\gamma = \frac{F}{l} \quad [\text{N/m}]$$
Measuring Surface Tension

\[ F = \gamma (2l) \]

Film surfaces

Wire

Side view

http://www.funsci.com/fun3_en/exper2/exper2.htm

http://cnx.org/contents/031da8d3-b525-429c-80cf-6c8ed997733a@8.9:83/College_Physics
Capillary and Surface Tension in Nature
Contact Angle and Capillarity

Capillary action:
- Wicking of liquids into small channels or porous media (or repulsion from).
- The action depends on the solid/liquid/gas system.

Contact angle: $\theta_c$
- Wetting ($\theta_c < 90^\circ$) vs. non-wetting ($\theta_c > 90^\circ$)
Contact Angle

- Surface tension (force per length)
- Angle is determined by the balance of forces at the point of interface
Contact Angle

- Surface tension (force per length)
- Angle is determined by the balance of forces at the point of interface
Drop thickness

- A drop of fluid on a surface

\[ h = \frac{\sqrt{2\gamma(1 - \cos \theta)}}{g \rho} \]

- \( h = \) height, \( g = \) gravity = 980 cm/sec\(^2\), \( \theta = \) contact angle, \( \rho = \) density
Forces - Capillary Effects

- A wetting fluid will rise in a capillary tube
- Equilibrium: pressure drop across meniscus

\[ h = \frac{2\gamma \cos(\theta)}{\rho gr} \]
Fluid ‘Creaping’
Capillary and Surface Tension in Nature
Hydrophobic Leaves

SEM micrographs of two hydrophobic leaves

Nelumbo nucifera (lotus)

Colocasia esculenta
Wettability and Roughness
Capillary Forces
Creating Surfaces with Different Surface Properties
Droplet on Nanostructured Surfaces
Creating nanostructures to study Droplet on Surfaces
Creating nanostructures to study Droplet on Surfaces
Droplet on Irregular Surfaces

\[ \cos_f(\varphi) = \frac{(f-1)}{(r-f)} \]

\( r \): roughness

\( f \): ratio of contact angle to the total horizon surface

Young’s critical angle \( \cos_f(\varphi) = \frac{(f-1)}{(r-f)} \)