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Formal and Factual Science (1935)

Rudolf Carnap

Notes.

1953j. Translation of "Formalwissenschaft und Realwissenschaft", 1935b.

1953-10. Formal and Factual Science. Trans. by Herbert Feigl and May Brodbeck. In *Readings in the Philosophy of Science*, Herbert Feigl and May Brodbeck, eds. New York: Appleton-Century-Crofts, [1953], pp. 123-128. Translation of Item 1935-2 with Literatur-Hinweise omitted.

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Exnotes: NO
Textnotes: NO

Table has been formatted. It needs its own page, but there needs to be a decision as to where it should be put.

"Sprache S_1" inconsistently translated as L_1 and S_1.

"kntradiktorisch" translated as "inconsistent".

"THEORY OF SCIENCE" MAY BE TAKEN to cover all investigations having science itself as their subject matter. Such investigations can be undertaken from different perspectives. Accordingly, we may distinguish psychological, sociological, historical or logical analyses of science, without necessarily separating them in practice. The logical analysis of science, called for short "logic of science," as I see it may be more precisely characterized as the logical syntax of the language of science.[i]

[i] Note † in Feigl and Brodbeck: According to Carnap's later views, semantical analysis must supplement the purely syntactical approach. Editors' note.

As an example of a problem in the logic of science, we shall deal in what follows with the problem of the relationship between two major fields of science, namely, the *formal sciences* (logic, including mathematics) and the *factual sciences* (embracing the totality of all empirical disciplines: physics, biology, psychology, sociology, history, etc.). The problem is here taken as one in the *logic* of science: such questions as, for instance, the psychological differences between the activities of research in the two fields shall not be discussed. Only the question concerning the *logical* relations between the two fields, that is, the difference in the syntactical character of their statements and statement-systems is our concern. While in their psychological character there is only a difference of degree and not of kind between the two fields, from a logical point of view a precise and fundamental difference can be demonstrated. This is based upon the syntactical *difference between analytic and synthetic statements.*

In order to show how the delimitation of the formal from the factual sciences can be achieved, we assume that the syntactical structure of the language of science is fully established. This requires the specification of a system of syntactical rules: first, the formation rules of the language in question, that is, the rules which determine the admissible forms of statements; second, the transformation rules, that is, the rules which determine under what conditions a statement is a consequence of other statements. (There are two different possibilities for the reconstruction of the language of science: the transformation rules may be restricted to logicomathematical rules or one may include extra-logical transformation rules. | We shall here, 124^t
for the sake of simplicity, restrict ourselves to the first kind.) Statements, according to their syntactical character and with regard to transformation rules, may be classified in the following way: a statement of the pertinent language will be called *analytic* if it is unconditionally valid according to the transformation rules and independently of the truth or falsity of other statements. The exact definition can, however, not contain such expressions as "unconditionally valid" but only the concept of "consequence", which is defined by the transformation rules. We define: a statement is analytic if it is a consequence of the null class of statements. Further, we call a statement *inconsistent* (self-contradictory) if it is unconditionally invalid; more precisely: if every statement of the language is a consequence of it. We call a statement *determinate* if it is either analytic or inconsistent. We call a statement *synthetic* if it is neither analytic nor inconsistent.

We arrive at a different classification of statements through a classi-

fication of signs. Signs having a logico-mathematical significance as, for example, 'or', 'every', 'not', '3', we shall call *logical signs;* signs with an extra-logical significance, such as, 'large', 'house', 'anger', we shall call *descriptive signs.* This difference can also be purely syntactically defined, that is, without reference to the significance of the signs. The class of logical signs may be characterized by saying that every statement containing only signs of this class is determinate. By a *logical statement* we shall understand one containing only logical signs; by a *descriptive statement* we shall mean one which contains at least one descriptive sign. Note, however, that while all synthetic statements are descriptive, the converse does not hold. The range of descriptive sentences is wider than that of synthetic statements.

The distinction between the formal and the factual sciences consists then in this: the first contains only analytic, while the second also contains synthetic statements. These relations are shown more clearly in the adjoining table, which represents diagrammatically the division among statements. The concepts "true" and "false"[ii] are added only for the sake of elucidation. They are not logical concepts definable in a purely syntactical manner. Synthetic statements constitute the core of science. They serve in the formulation of possible states of affairs (both actual and non-actual). In addition, we have analytic statements arranged in three stages: first, the descriptive analytic statements; they are still in close relation to the factual sciences inasmuch as they contain descriptive signs, that is, signs of extra-logical entities. But they contain them in a form that permits the question as to whether such statements are true or false to be answered independently of the nature of these entities, namely, merely on the basis of the transformation rules of language. Further, there are the analytical logical statements. Among them we may distinguish those which are logical in the narrower sense of that word and the mathematical statements. There is no fundamental difference between the mathematical statements and the
126^t other | purely logical ones; but one may for practical purposes distinguish them by the decision to call "mathematical" only those statements which contain numerals or predicates relating to numerals, etc.

Since synthetic statements are sufficient for the formulation of any particular assertion as well as of general laws, it is possible to reconstruct the language of science in such a manner that it contains only synthetic statements. This need not diminish the content of science. Given some language L_1 for the whole of science, then language L_2 can be constructed

[ii] single quotes in German

as follows: the formation rules for L_2 stipulate that all synthetic statements of L_1 and only these are admitted as statements in L_2. The transformation rules for L_1 will be established in the following way: those transformation rules of L_1 which have the form of ordinary rules of inference (i. e., transformation rules with premises) are accepted without alteration in L_2. However, the transformation rules of L_1 which have the form of axioms (i. e., transformation rules with an empty class of premises) and perhaps a few other analytic statements of L_1 are replaced by corresponding transformation rules (with premises) in L_2. For example, instead of the axiom "$P \supset (P \vee Q)$"[iii] in L_1 we adopt in L_2 the following transformation rule: "Every statement of the form $S_1 \vee S_2$ is a direct consequence of S_1."[iv] In L_2 there are then no longer any purely logical statements; what they achieve in L_1 is achieved in L_2 by corresponding rules. Similarly, "$2 + 2 = 4$" is a sentence in L_1 but not in L_2. In its place we have in L_2 the derived rule: "The expressions '2 + 2' and '4' are always mutually substitutable." ⟦substitutable. CT⟧ L_2 then contains neither logic nor mathematics as statement-systems. But not only is every particular factual statement and every law of L_1 contained in L_2; also every logical deduction, including all mathematical computations and transformations, which in L_1 leads from synthetic premises to synthetic conclusions can be carried out in L_2. However, the procedure is then not as simple as in L_1; hence, the form of language L_2, while possible, is *inexpedient.* Therefore, we shall prefer the form L_1, which results from L_2 by adding to the synthetic statements certain auxiliary statements, namely, the analytic (and inconsistent) ones. These auxiliary statements have indeed no factual content or, to speak in the material idiom, they do not express any matters of fact, actual or non-actual. Rather they are, as it were, mere calculational devices, but they are so constructed that they can be subjected to the same rules as are the genuine (synthetic) statements. In this way they are an easily applicable device for operations with synthetic statements.

There is moreover the possibility of a compromise, namely, to include in a language besides the synthetic statements some but not all analytic statements contained in the present day language of science. But this procedure is probably even less expedient than that of the complete elimination of analytic statements. Once statements devoid of factual content are admitted at all, there is no cogent reason why their advantages should not be utilized to the fullest extent. The proposal to admit only the de|scriptive analytic 127^t

[iii] Single quote marks in German

[iv] S_i set in Fraktur in German.

statements might seem defensible. After all, they are more closely related to synthetic statements than the rest of the analytic statements: their elimination would have the unpleasant consequence that the compounding of two statements as by '$\vee$' ("or") would not always yield a statement as, for example, the compound "S_1 or not-S_1". (This would render the formation rules indefinite, thus involve grave disadvantages.) The most inexpedient proposal, however, would be a delimitation according to which logical statements in the narrower sense are admitted in the language but the mathematical ones are excluded. (In this manner Wittgenstein, for example, drew the line of demarcation; but he formulated this formation rule not as a proposal but as an assertion about "*the* language".)

Science uses synthetic and analytic statements in the following manner. The factual sciences establish synthetic statements, e. g., singular statements for the description of observable facts or general statements which are introduced as hypotheses and used tentatively. From the statements thus established the scientists try to derive other synthetic statements, in order, for instance, to make predictions concerning the future. The analytic statements served in an auxiliary function for these inferential operations. All of logic including mathematics, considered from the point of view of the total language, is thus no more than an auxiliary calculus for dealing with synthetic statements. *Formal science* has no independent significance, but is an auxiliary component introduced for technical reasons in order to facilitate linguistic transformations in the *factual sciences.* The great importance of the formal sciences, that is, of logic and mathematics, within the total system of science is thereby not in the least denied but instead, through a characterization of this special function, emphasized. This characterization of the logical function of the formal sciences as an auxiliary calculus is not incompatible with the psychological fact that it is certainly not always, and perhaps only in infrequent cases, that the motive for research in any branch of the formal sciences is possible application in the factual sciences. The question as to whether the calculus of analytic statements, in view of the above characterization, is still to be called a science or not is a relatively unimportant terminological issue. In keeping with the customary use of terms, it may be better to designate as sciences all systems with deductive connections; hence to include therein logic and mathematics and not only systems of synthetic statements, that is, those of the factual sciences.

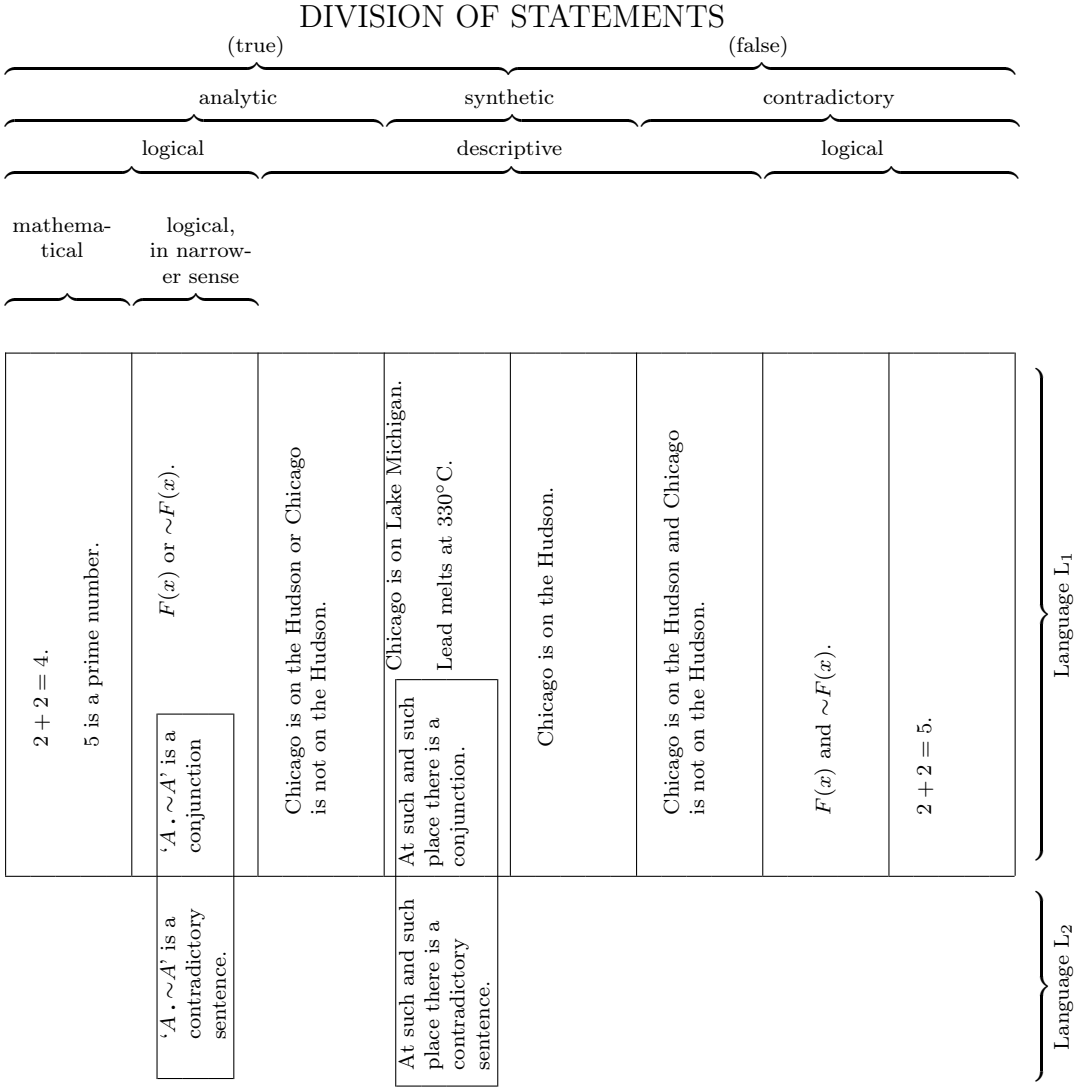
A remark may be added concerning the question as to where in our scheme *syntax* is to be located. The scheme represents a division of the

statements of a definite language, such as S_1. Now if statements of pure syntax concerning the statement forms of this language are formulated (e. g., "A sentence of such-and-such form is analytic in S_1", "Two statements of such-and-such form are incompatible in S_1"), then this would generally be done in a second language S_2. In this case syntax lies entirely outside of our scheme, namely, in the logico-mathematical part of the | language S_2. It is, 128^t
however, also possible to formulate syntactical statements about statements of S_1 in S_1 itself. They then belong either to the logical statements in the narrower sense, or — if the syntax is arithmetized — to the mathematical ones. But this formulation in S_1 would not be possible for all syntactical statements; some concepts referring to S_1 (e. g., "analytic in S_1", "contradictory in S_1") cannot be defined by means of the concepts of S_1 itself but only in a richer language S_2. — What has just been said is true of pure syntax; descriptive syntax, which deals with sentences as physical objects, belongs to the factual sciences.

In adjoining the formal sciences to the factual sciences *no new area of subject matter* is introduced, despite the contrary opinion of some philosophers who believe that the "real" objects of the factual sciences must be contrasted with the "formal", "*geistig*" or "ideal" objects of the formal sciences. *The formal sciences do not have any objects at all;* they are systems of auxiliary statements without objects and without content. The emphasis on a sharp delimitation between the formal and the factual sciences therefore leaves the *unity of science* unaffected.

Pointers to the Literature:

On logical syntax and the logic of science: R. Carnap, Logische Syntax der Sprache. Wien, 1934 [[Carnap 1934f]]. — Shorter, more accessible presentations: R. Carnap, Die Aufgabe der Wissenschaftslogik. Wien, 1934 [[Carnap 1934e]]. R. Carnap, Philosophy and Logical Syntax. London, 1934 [[Carnap 1935a]] — On the impossibility of defining certain syntactic concepts in S_1 itself: R. Carnap, Die Antinomien und die Unvollständigkeit der Mathematik. Monatsh. Math. Phys. 41, 1934 [[Carnap 1934]]. These considerations are based on the results of: K. Gödel, Über formal unentscheidbare Sätze ... Monatsh. Math. Phys. 38, 1931 [[Gödel 1931]].


DIVISION OF STATEMENTS
(true)
(false)
analytic
synthetic
contradictory
logical
descriptive
logical
mathematical
logical, in narrower sense
$2+2=4$.
5 is a prime number.
$F(x)$ or $\sim F(x)$.
'$A\,.\sim A$' is a conjunction
'$A\,.\sim A$' is a contradictory sentence.
Chicago is on the Hudson or Chicago is not on the Hudson.
Chicago is on Lake Michigan.
Lead melts at 330°C.
At such and such place there is a conjunction.
At such and such place there is a contradictory sentence.
Chicago is on the Hudson.
Chicago is on the Hudson and Chicago is not on the Hudson.
$F(x)$ and $\sim F(x)$.
$2+2=5$.
Language L_1
Language L_2
DIVISION OF SCIENCES
pure syntax
descriptive syntax
Mathematics
Logic in narrower sense
Applied Logic
Logic in Broader sense
Formal Science
Factual Science

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4:15 substitutable."] substi-
tutable. CT

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Cited in 1932d-en, 1934a, 1934e, 1934g, 1935a, 1935b-en, 1935c, 1936c, 1936j, 1937d, 1938a, 1938b, 1938c, 1939a, 1940a. (`Carnap1934f` / Carnap 1934f)

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Cited in 1935b-en, 1935c, 1936c, ,1936h-fr, 1936j, 1938b, 1938c (`Carnap1935a` / Carnap 1935a)

Gödel, Kurt. 1931. "Über formal unentscheidbare Sätze der *Principia Mathematica* und verwandter Systeme I." *Monatshefte für Mathematik und Physik* 38(1), pp. 173–198.
Cited in 1929e, 1934e, 1934f, 1935b-en, 1939a, 1956c. Entry Checked (`Godel1931` / Gödel 1931)

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