Fluids – Basic Principles

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Topics

• Fluids and Solids
• Type of Flow
• Reynolds number
• Diffusion
• Surface Tension
• Capillary Forces
Fluids

- **Fluid**
  - Has volume, but no shape
  - Any substance that deforms continuously under the application of shear
  - Fluid behaves as a continuum
  - Fluid ‘sticks’ to surfaces (no-slip condition)

- **Microfluidics** is the fluid dynamics in microscopic domains and very small volumes
Basic Properties

- Type of Fluids
  - Newtonian
  - non-Newtonian
- Fluid Flow
  - Laminar Flow
  - Turbulence
- Parameters
  - Shear
  - Viscosity
  - Surface Tension
Viscosity is a measure of ?? of the fluid to the flow

Symbols: \( \eta \) or \( \mu \)
Viscosity is a measure of resistance (friction) of the fluid to the flow

This determines “flow rate”

Symbols: \( \eta \) or \( \mu \)
\[ \tau = \mu \frac{du}{dy} \]

\[ \tau = \frac{F}{A} \]
Non-Newtonian Fluids

\[ \tau = \mu \frac{du}{dy} \]

\[ \left( \tau = \frac{F}{A} \right) \]
Viscosity ($\mu$)

- “inverse fluidity”, “measure of fluid resistance to flow”
- Ratio of the shear stress to the strain rate:

$$\tau = \mu \frac{du}{dy}$$

- Units: Poise “P” (g/cm·s) or Pa·s
- Dependent on temperature:
  - Liquids: $\mu \downarrow$ as $T \uparrow$
  - Gases: $\mu \uparrow$ as $T \uparrow$
Viscosity (\(\mu\))

- **Constant**
  - Only for Newtonian Fluids
    \[ \tau = \mu \frac{du}{dy} \]
  - Water, ethanol, gasoline
- **Non-Newtonian Fluids**
  - Bingham Plastic
    - Ketchup, peanut butter
  - Pseudo-plastic (shear-thinning)
    - Latex paint, blood
  - Dilatant (shear-thickening)
    - Starch solution, AWD coupling fluid
Example: Dilatant Fluid

“Walking over water (plus some corn starch)!”

(Steve Spangler on The Ellen Show)

http://www.youtube.com/watch?v=RUMX_b_m3Js
Summary
Newtonian Foods

**Examples:**
- Water
- Milk
- Vegetable oils
- Fruit juices
- Sugar and salt solutions
Pseudoplastic (Shear thinning) Foods

Examples:
- Applesauce
- Banana puree
- Orange juice concentrate
- Oyster sauce
Dilatant (Shear thickening) Foods

Examples:
• Liquid Chocolate
• 40% Corn starch solution

Shear stress
Shear rate
Examples:

- Tooth paste
- Tomato paste
Fluid Movement

- Gravity pumped
- Vacuum pumps
- Electro-osmotic Flow
- Thermal Flow
Fluids - Types of Flow

- Laminar Flow (Steady)
- Turbulent Flow
- Laminar Flow ---> Turbulent Flow
Fluids - Types of Flow

- **Laminar Flow** (Steady)
  - Energy losses are dominated by viscosity effects
  - Most Flow in MEMS are Laminar

- **Turbulent Flow**
  - Most flow in nature are turbulent!

- Laminar Flow ---> Turbulent Flow
  
  Reynolds Number Re
  
  Re is a measure of turbulence
Reynolds number (Re) = inertial forces / viscous forces

Re = Kinetic energy / energy dissipated by shear
Implies inertia relatively important

\[ Re = \frac{\rho V_D L}{\eta} \]

\( V_D = \text{Drag velocity}, \)
\( L = \text{characteristic length}, \)
\( \eta = \text{viscosity}, \)
\( \rho = \text{density} \)
Reference: Van Dyke, *Album of Fluid Motion*
Flow Regimes

- **Laminar Flow**
  - Viscous forces dominate ($\mu$).
  - Observed in small conduits or at low flow rates.
  - Like concentric liquid cylinders in a pipe.

- **Turbulent Flow**
  - Inertial forces are significant ($\rho$).
  - Observed in large conduits or at high flow rates.
  - Unpredictable flow due to vortices, eddies and wakes.

- **Transitional Flow**
  - Generally turbulent in the center of the channel and laminar near the surfaces.
Example: Mixing Viscous Liquids

- Laminar Flow: high viscosity (corn syrup) and relatively low fluid velocity, low $Re$

$$Re = \frac{\rho V D L}{\eta}$$

http://www.youtube.com/watch?v=p08_KITKP50
Newton’s Second Law for Fluidics

Newton’s 2nd Law
Time rate of change of momentum of a system equal to net force acting on system

\[ \Sigma F = \frac{dP}{dt} \]

In Fluids

??? equation
Newton’s Second Law for Fluidics

Newton’s 2nd Law

Time rate of change of momentum of a system equal to net force acting on system

$$\sum F = \frac{dP}{dt}$$

In Fluids

Navier-Stokes equation

Sum of forces acting on a control volume = Rate of momentum efflux from control volume
Navier - Stokes Equation

\[ \rho \frac{dU}{dt} = -\nabla P + \rho g + \eta \nabla^2 U + \frac{\eta}{3} \nabla(\nabla \cdot U) \]

U = Velocity
Navier - Stokes Equation

\[ \rho \frac{dU}{dt} = -\nabla P + \rho g + \eta \nabla^2 U + \frac{\eta}{3} \nabla (\nabla \cdot U) \]

For noncompressible Fluid \( \nabla \cdot U = 0 \)

\[ \rho \frac{dU}{dt} = -\nabla P + \rho g + \eta \nabla^2 U \]

If inertial forces dominate inertial forces dominate due to small dimensions, even though velocity can be high

\[ \frac{dU}{dt} = -\frac{1}{\rho} \nabla P \]

\( U = \text{Velocity} \)

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Laminar Flow:
Viscous interaction between the wall and the fluid is strong, and there is no turbulences or vortices.

\[ N_R \ll 1, \text{ laminar flow} \]
Poiseuille Flow

Poiseuille flow profile.

Average \( U = \frac{h^2 \nabla P}{12 \eta L} = \frac{2}{3} U_{\text{max}} \)
Pressure Driven
Diffusion

Stokes - Einstein Equation

Diffusion of a particle (gas, fluid)

Translational Diffusivity

Rotational Diffusivity

\[ D_t = \frac{K_B T}{6\pi \eta a} \]

\[ D_r = \frac{K_B T}{8\pi \eta a^3} \]

\( a = \text{particle radius} \)

\( \eta = \text{viscosity} \)
Flow Profiles

Without diffusion

With diffusion
Fluid Flow in very narrow channels

Laminar flow in narrow channels leads to no “mixing” of the fluids - Good or Bad?
Fluid Flow in very narrow channels

Diffusion $\sim 1/a$

Can use the “T” channel as a

$D_t = \frac{K_B T}{6\pi \eta a}$

$D_r = \frac{K_B T}{8\pi \eta a^3}$