Newton's Second Law

\[ \vec{F}_{\text{net}} = m \vec{a} \]

- Problem-Solving Strategy:
  1. Draw a picture; draw and label all forces present.
  2. Choose a coordinate system.
  3. Write Newton's 2nd Law in each direction.
  4. Solve for unknown quantities.
  5. Plug in numbers last.

Forces

- Gravitational Force \( F_g \)
  - Force due to gravity; equal to an object's mass multiplied by the gravitational acceleration \( g = 9.81 \text{ m/s}^2 \)
  \[ F_g = mg \]

- Normal Force \( N \)
  - Contact force that always acts perpendicular to a surface.
  - Magnitude equal to whatever is necessary in order to keep the object from falling through the surface.

- Tensile Force \( T \)
  - Pulling force that acts in the direction of the string and is the same at every point on the string.
**Friction**

- Static Friction $f_s$: friction force that resists the motion of two bodies with respect to each other.
  
  maximum value: $f_s, \text{max} = \mu_s N$

- Kinetic Friction $f_k$: friction force that resists the motion of two bodies with respect to each other.
  
  $f_k = \mu_k N$

**Drag**

- Viscous Drag: drag force for small objects moving slowly through viscous fluids.
  
  $F_{\text{drag}} = 6 \pi \eta RV$
  
  $\eta$ - viscosity of fluid
  $R$ - radius
  $v$ - speed

- Pressure Drag: drag force for large objects moving quickly through non-viscous fluids.
  
  $F_{\text{drag}} = \frac{1}{2} CD \rho A v^2$
  
  $\rho$ - density of fluid
  $A$ - cross-sectional area
  $v$ - speed
  $CD$ - drag coefficient
1. Two blocks 1 and 2, on a frictionless table, are pushed from the left by a horizontal force $F_1$ and on the right by a horizontal force $F_2$, as pictured. The magnitudes of the pushing forces satisfy the inequality $|F_1| > |F_2|$

Which of the following statements is true about the magnitude of the contact force $N$ between the two blocks?

A) $N > |F_1| > |F_2|$
B) $|F_1| > N > |F_2|$
C) $|F_1| > N = |F_2|$
D) $|F_1| = N > |F_2|$
E) $|F_1| = |F_2| > N$

Because $|F_1| > |F_2|$, the two blocks will move to the right.

By Newton's 3rd law, the force of block 1 on block 2 is equal to the force of block 2 on block 1, this is the "contact force" between the two blocks.
2. You push on a block of mass M with a horizontal force \( F \), as shown below. A block of mass \( m \) on top of M moves precisely along with it. What force directly causes \( m \) to accelerate horizontally along with M?

![Diagram of forces](image)

A) The normal force between the blocks  
B) The static friction force between the blocks  
C) The kinetic friction force between the blocks  
D) The gravitational force on \( m \)  
E) The force you apply on M  
F) No force is required because the masses are in contact

→ The static friction force keeps \( m \) from losing contact with M and directly causes \( m \) to accelerate horizontally along with M.

*The normal and gravitational forces act in the vertical direction; the kinetic friction force would appear if \( m \) was moving with respect to M; the force you apply on M does not directly affect \( M \).

What is the maximum magnitude of \( F \) in order for \( m \) to move along with M? How would kinetic friction on the surface under M change this?

→ First, let's draw a free body diagram and write Newton's second law for \( m \):

![Free body diagram of \( m \)]

\[ F_{net, x} = ma \]
\[ F_{net, y} = 0 \]
\[ N_m - mg = 0 \]
\[ N_m = mg \]

**NOTE:** \( F \) points to the right since, without it, \( m \) would move to the left. The friction force acts in the direction opposite to this movement.
2.

- Because we are trying to solve for the maximum magnitude of \( F \), the friction force \( F_s \) will also be at its max value:

\[
F_s = \mu_s N
\]

\[
F_s = \mu_s mg
\]

- The two blocks are moving together, so they move at the same acceleration, which can be solved for by:

\[
F_s = ma
\]

\[
\mu_s mg = ma
\]

\[
a = \mu_s g
\]

- Now let's draw a free body diagram and write Newton's second law for \( M \):

\[
\begin{align*}
M & \quad \text{\( x \)-direction} \\
F - F_s &= Ma \\
F &= (M + m)\mu_s g
\end{align*}
\]

\[
\begin{align*}
N_M &= Mg + N_m \\
N_m &= Mg + mg \\
N_m &= (M + m)g
\end{align*}
\]

\text{Note: Block M has two normal forces: \( N_M \) due to the ground and \( N_m \) due to block m.}

\text{Also, \( F_s \) now points to the left since block M is moving to the right.}

- We have already solved for \( F_s \) and \( a \), so we can now determine the maximum magnitude of \( F \):

\[
F = F_s + Ma
\]

\[
F = (M + m)\mu_s g
\]

\text{Maximum magnitude, frictionless table.}
2. Now, if the table was not frictionless, a kinetic friction force would also be applied to block $M$; in this case, the free body diagram and Newton's second law become

\[
\begin{align*}
\text{x-direction} & \\
F_{net,x} &= Ma \\
F - f_k &= Ma \\
F &= Ma + f_k + f_k
\end{align*}
\]

\[
\begin{align*}
\text{y-direction} & \\
F_{net,y} &= 0 \\
N_m - N_m - Mg &= 0 \\
N_m &= N_m + Mg \\
F_k &= \mu_k N_m \\
F_k &= \mu_k (M+m)g
\end{align*}
\]

The acceleration and static friction force remain unchanged; the kinetic friction force is equal to

\[f_k = \mu_k (M+m)g\]

We can now solve for the maximum magnitude of $F$, in the presence of kinetic friction

\[
F = Ma + f_k + f_k
\]

\[
= M(Mg) + \mu_k mg + \mu_k (M+m)g
\]

\[
= \mu_k (M+m)g + \mu_k (M+m)g
\]

\[
F = (\mu_k + \mu_k)(M+m)g
\]

Maximum magnitude, kinetic friction present

As, the maximum magnitude of $F$ increases in the presence of kinetic friction
3. If a 70-kg skier is subjected to a pressure drag force \( F_{\text{drag}} = \frac{1}{2} C_d \rho s A v^2 \), with \( C_d = 0.5 \), \( \rho = 1.2 \text{ kg/m}^3 \), and \( A = 0.5 \text{ m}^2 \), and is also subjected to a kinetic friction force with \( \mu_k = 0.1 \), calculate the terminal velocity for the skier on a slope that is inclined at \( \theta = 30^\circ \) relative to the horizontal.

For problems involving an inclined plane, it is most convenient to use a tilted coordinate system that is aligned with the plane.

We can solve for the terminal velocity by drawing a free body diagram and writing Newton's second law:

\[
\begin{align*}
\text{x-direction:} \\
mg \sin \theta - f_k - F_{\text{drag}} &= 0 \\
F_{\text{drag}} &= mg \sin \theta - f_k \\
\frac{1}{2} C_d \rho s A v^2 &= mg \sin \theta - \mu_k N \\
v^2 &= \frac{2 (mg \sin \theta - \mu_k mg \cos \theta)}{C_d \rho s A} \\
V &= \sqrt{\frac{2 mg (\sin \theta - \mu_k \cos \theta)}{C_d \rho s A}} \\
&= \sqrt{\frac{2 (70 \text{ kg})(9.81 \text{ m/s}^2)(\sin(30^\circ) - (0.1) \cos(30^\circ))}{(0.5)(1.2 \text{ kg/m}^3)(0.5 \text{ m}^2)}} \\
V &= 43.5 \text{ m/s}
\end{align*}
\]