CMSC 132: Object-Oriented Programming II

Graph Implementation

Department of Computer Science
University of Maryland, College Park
Graph Implementation

How do we represent edges?

- Adjacency matrix
  - 2D array of neighbors
- Adjacency list
  - List of neighbors
- Adjacency set / map
  - Set / map of neighbors

Important for very large graphs

- Affects efficiency / storage
Adjacency Matrix

- **Representation**
  - 2D array
  - Position $j, k \Rightarrow \text{edge between nodes } n_j, n_k$

- **Example**

```
2 3 5
1 1 8
3 3 3
4 1.5
5
```

```
\[
\begin{array}{cccccc}
\text{1} & \text{2} & \text{3} & \text{4} & \text{5} \\
1 & 3.7 & \text{#} & \text{#} & \text{#} \\
2 & \text{#} & 3.7 & \text{#} & \text{#} \\
3 & \text{#} & \text{#} & 10.2 & \text{#} \\
4 & \text{#} & \text{#} & \text{#} & 1.0 \\
5 & \text{#} & \text{#} & \text{#} & \text{#} & \text{#} \\
\end{array}
\]
```
Adjacency Matrix

Representation (cont.)
- Single array for entire graph
- Undirected graph
  - Only upper / lower triangle matrix needed
  - Since \( n_j, n_k \) implies \( n_k, n_j \)
- Unweighted graph
  - Matrix elements \( \Rightarrow \) boolean
- Weighted graph
  - Matrix elements \( \Rightarrow \) weight
**Adjacency List/Set**

**Representation**

- For each node, store
  - List/Set of neighbors / successors
    - Linked list
    - Array list

- For weighted graph
  - Also store weight for each edge
  - Using a Map is a good choice

- For undirected graph with edge \((a \leftrightarrow b)\)
  - Nodes \(a\) & \(b\) need to store each other as neighbor

- For directed graph with edge \((a \rightarrow b)\)
  - Node \(a\) needs to store node \(b\) as neighbor
Adjacency List

Example

Unweighted graph

node 1: {2, 3}
node 2: {1, 3, 4}
node 3: {1, 2, 4, 5}
node 4: {2, 3, 5}
node 5: {3, 4, 5}

Weighted graph

node 1: {2=3.7, 3=5}
node 2: {1=3.7, 3=1, 4=10.2}
node 3: {1=5, 2=1, 4=8, 5=3}
node 4: {2=10.2, 3=8, 5=1.5}
node 5: {3=3, 4=1.5, 5=6}
Adjacency Set / Map

Representation

- For each node, store
  - Set or map of neighbors / successors
- For unweighted graph
  - Use set of neighbors
- For weighted graph
  - Use map of neighbors, with value = weight of edge
- For undirected graph with edge \((a \leftrightarrow b)\)
  - Nodes a & b need to store each other as neighbor
- For directed graph with edge \((a \rightarrow b)\)
  - Node a needs to store node b as neighbor
Graph Space Requirements

- Adjacency matrix
  - \(\frac{1}{2} N^2\) entries (for graph with \(N\) nodes, \(E\) edges)
  - Many empty entries for large, sparse graphs

- Adjacency list
  - \(2 \times E\) entries

- Adjacency set / map
  - \(2 \times E\) entries
  - Space overhead per entry
    - Higher than for adjacency list
Graph Time Requirements

- **Adjacency matrix**
  - Can find individual edge \((a, b)\) quickly
  - Examine entry in array \(\text{Edge}[a,b]\)
    - Constant time operation

- **Adjacency list / set / map**
  - Can find all edges for node \((a)\) quickly
  - Iterate through collection of edges for \(a\)
    - On average \(E / N\) edges per node
### Graph Time Requirements

**Average Complexity of operations**

For graph with \( N \) nodes, \( E \) edges

<table>
<thead>
<tr>
<th>Operation</th>
<th>Adj Matrix</th>
<th>Adj List</th>
<th>Adj Set/Map</th>
</tr>
</thead>
<tbody>
<tr>
<td>Find edge</td>
<td>( O(1) )</td>
<td>( O(E/N) )</td>
<td>( O(1) )</td>
</tr>
<tr>
<td>Insert edge</td>
<td>( O(1) )</td>
<td>( O(E/N) )</td>
<td>( O(1) )</td>
</tr>
<tr>
<td>Delete edge</td>
<td>( O(1) )</td>
<td>( O(E/N) )</td>
<td>( O(1) )</td>
</tr>
<tr>
<td>Enumerate edges for node</td>
<td>( O(N) )</td>
<td>( O(E/N) )</td>
<td>( O(E/N) )</td>
</tr>
</tbody>
</table>
Choosing Graph Implementations

- **Graph density**
  - Ratio edges to nodes (dense vs. sparse)

- **Graph algorithm**
  - **Neighbor based**
    - For each node X in graph
      - For each neighbor Y of X  // adj list faster if sparse
        - doWork()
  - **Connection based**
    - For each node X in ...
      - For each node Y in ...
        - if (X,Y) is an edge  // adj matrix faster if dense
          - doWork()