Ch. 5.4 Grasp Planning

1. Force Closure and Form Closure

"Form Closure = A sub-class of force closure that doesn’t rely on friction ($\mu = 0$). [Proposition 5.3]"

Form Closure implies Force Closure.

Assumptions about the object $O$:

- Rigid solid
- Exactly known geometry
- "Regular:" Constructive solid geometry (closed, compact) where $O$ is the closure of its interior ("well-behaved" objects)
- $O \subseteq \mathbb{R}^3$

Let the object coordinate frame $\{O\}$ origin be at center of mass.

Let $\Sigma = \partial(O)$ be the boundary of $O$ – connected, piecewise smooth.

Given a set of $n$ contact points $C = \{c_i\} i = 1 \ldots n c_i \in \Sigma$

Let $\Lambda(\Sigma)$ be the set of wrenches that can be applied to $O$ with frictionless point contacts ($\mu = 0$)

$p_{ci} =$ location of $c_i$ in $\{O\}$

$n_{ci} =$ inward normal at $c_i$
\[ \Lambda(\Sigma) = \left\{ \left[ p_{cl} \times n_{cl} \right] \right\} \]

\( F(O, C) \) is true if C puts O into Form/Force Closure

*If the convex hull of \( \Lambda(\Sigma) \) contains the origin \( \{O\} \), then \( F(O, C) \).*

Also, let \( p \) be the *dimension of the wrench space*, \( \mathbb{R}^p \).

- \( p = 3 \) in the plane
- \( p = 6 \) in space (3D)

Therefore, *if \( \Lambda(\Sigma) \) positively spans \( \mathbb{R}^p \), then \( F(O, C) \).*

**Exceptional Surface:**

Object \( O \) with boundary \( \partial(O) \) such that it cannot be grasped (without friction).

Examples: sphere, circle, cylinder, etc.

\[ \neg \exists \subset F(O, C) \]

**II. Grasp Planning in the Plane (\( \mu = 0 \)) – Geometric Intuition**
\( \mathcal{F}(O, C) \)?

**Analysis:** Given O, C: is \( \mathcal{F}(O, C) \) true?

**Synthesis:** Given O, find C such that \( \mathcal{F}(O, C) \) is true.

I. **Rigid Body Motion:**

\[
\xi = \begin{bmatrix} 2y \\ -2x \\ 1 \end{bmatrix} \quad \text{(pure rotation)} \quad \text{or} \quad \begin{bmatrix} v_x \\ v_y \\ 0 \end{bmatrix} \quad \text{(pure translation)}. 
\]

Pure translation:

Intersection at \( \infty \).
II. **Frictionless point contact: constraints on $\xi$**

Define $\text{sgn}(\xi) = \begin{cases} 
+1 & \text{if } \theta > 0 \\
0 & \text{if pure translation} \\
-1 & \text{if } \theta < 0 
\end{cases}$

III. **Rotation Center Locus: Multiple (frictionless) Contacts**:
C = \{c_1, c_2, c_3, c_4\}

Let \( \Xi = \{\xi_i\} \)

If \( \Xi = \emptyset \): \( \mathcal{F}(O, C) \)

To eliminate the locus: place \( p_4 \) anywhere on the jagged edges.

**Are 3 contacts enough?**

Is O in form closure? **No**: The locus is a point with \( \text{sgn}(p) = \pm 1 \), so there is *infinitesimal* rotation.

\[ \therefore \text{Need} \geq 4 \text{ contacts in the plane.} \]

So far, we haven’t been allowing contacts on the corners.

Contacts at concave vertices \( \rightarrow \) very important, there is a lot of constraint there.
2\textsuperscript{nd} Order Form Closure:

IV. \textbf{Number of Required Contacts:}

Given a set of vectors $X = \{v_1, \ldots, v_k\}$, $X$ positively spans $\mathbb{R}^p$ if and only if $\text{co}(X)$ [Convex-hull of $X$] contains a neighborhood of the origin (pg. 255).

\textbf{Theorem 5.4: (Caratheodory) 1911 (Greek)}

At least $p+1$ vectors are necessary to positively span $\mathbb{R}^p$.

For $p = 2$: $\forall \ v_1, v_2: -(v_1 + v_2)$ is outside the positive span of $v_1, v_2$
Theorem 5.5: (Steinitz – Jewish German)

Given a set of vectors that positively span \( \mathbb{R}^p \), \( \exists \) a subset of 2p or Fewer sufficient to positively span \( \mathbb{R}^p \).

Recall: \( p \) = dimension of the wrench space

<table>
<thead>
<tr>
<th>Space</th>
<th>Object type</th>
<th>Lower</th>
<th>Upper</th>
<th>FPC</th>
<th>PCWF</th>
<th>SF</th>
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<td>Polyhedral</td>
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<td>4</td>
</tr>
</tbody>
</table>

V. **Grasp planning in the plane \((\mu > 0)\)**

2 point contacts with friction (2PCWF) in plane.

Define **grasp axis**: \([p_1 - p_2]\)

\[
\tan(\alpha) = \mu
\]
Theorem 5.6: PCWF

G is in FC if and only if the grasp axis lies strictly inside both friction cones. [Nguyen ‘88]

(Related to Def. 5.2, Prop. 5.1, 5.2, 5.3)

Extends to 2 point contacts with friction in 3D.

Theorem 5.7: Check both contacts individually. [Nguyen ‘88]

Check 1: Is \( c_1 \) inside Friction Cone 2?

Check 2: Contact 1 is stable if \( \frac{d_{n_2}}{d_{\mu_2}} < \mu \).

If both are stable, \( F(O, C) \).

i.e. Don’t need to approximate the friction cone.
VI. **Grasp Planning with Uncertainty (in Pose) in the Plane**

Assume: Known object

Pose: Not known precisely

**Parallel-Jaw Gripper (pg. 11 Problem)**

Dr. Ken Goldberg

Convex hull of O

**Radius Function and Diameter Function:**

**Squeeze function** $s, s' \rightarrow s'$

*Piecewise constant monotone step function*

$\theta_x$ where $\theta_x$ is leftmost point

Symmetry in object $\rightarrow$ periodicity $\rightarrow$ aliasing