CH 5.4 GRASP PLANNING

I. FORCE CLOSURE : Proposition 5.3

"FORC CLOSURE" : Special subclass of force closure that doesn't rely on friction.

The object : rigid solid with exact geometry

Connected : o C R^3 and "regular" (CSG)

Let object frame be at com : \{o\}

Let \Sigma = \partial o boundary of o : connected, piecewise smooth

Given \{c_i\} : Let \Lambda(\Sigma) set of all wrenches that can be applied to o w/ frictionless point contacts

with \mu = 0:

\Lambda(\Sigma) = \{ [nc_i] \}

For \mu = 0: force closure (and form closure):

\Phi(0,0) if:

To be graspable : convex hull of \Lambda(\Sigma) must contain the origin

Exceptional Surface : \Sigma such that \exists C

Cannot be grasped (w/ friction) eg, sphere, circle, cylinder...
GRASP PLANNING W/ MULTIPLE CONTACTS

Let $\Xi = \{ \xi_i \}$

Theorem: if $\Xi = \emptyset$: $FC(0,c)$

To eliminate this locus:

Place $P_4$ anywhere on lower right edge.

HW: Given $0 = (3,2),(-4,2),(-1,0),(-1,-5),(0,-5)$, construct a FC grasp using rotation center.

Are 3 enough?

$g_0(0,c)$?

Is point $0$ in form closure?

No: Locus is a point with $sgn(\phi) = \pm 1$

So infinitesimal rotation.

:: Need $\geq 4$ contacts in plane.

(BTW: yes: 2nd order form closure)

Local curvature of contact.
Prop. 5.3

Given a set of vectors $X = \{v_1, \ldots, v_k\}$, they positively span $\mathbb{R}^p$ iff $\text{Co}(X)$, the convex hull of $X$, contains the origin and a neighborhood of $p$.

Theorem 5.4: \((\text{card theory})\) 1911 (Greek)

At least $p+1$ vectors are necessary to positively span $\mathbb{R}^p$.

For $p=2$,

\[
\begin{align*}
V_1 & = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\
V_2 & = \begin{pmatrix} 0 \\ 1 \end{pmatrix}
\end{align*}
\]

$V_1, V_2 : -(V_1 + \theta V_2)$ is always outside the pos. span of $V_1, V_2$.

Theorem 5.5: \((\text{stelitz})\) Jewish German

Given a set of vectors that positively span $\mathbb{R}^p$, if a subset of $2p$ or fewer is sufficient to positively span $\mathbb{R}^p$.

RECALL $p = \text{dimension of wrench space}$

Table 5.3.
5.4.2 GRASP PLANNING IN THE PLANE ($\mu > 0$)

2 point contacts w/friction (PCWF) in plane

$P_1$ - $P_2$

define grasp axis: $[P_1 - P_2]$

Thm 5.6 PCWF

G will be in FC iff grasp axis lies strictly inside both friction cones

See p. 232-23 Fig 5.13

Extends to

2 point contacts w/friction in 3D space

Thm 5.7:

CHECK BOTH CONTACTS INDIVIDUALLY:

Check 1: $P_1$: INSIDE FC? easy to check.

Construct $n_2$

$dn_2$: distance from $P_1$ to $n_2$

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Contact at $P_1$ is stable if $dn_2 / dn_2 < \tan\mu$

ie. don't need to approx friction cone.
CHECK 2: \( P_2 \) inside \( FC_1 \)?

if both True, then 3D force closure

GRASP PLANNING w/uncertainty (in pose) in plane

ASSUME: KNOWN OBJECT

POSE: NOT KNOWN PRECISELY

Parallel Jaw Gripper


Convex hull to radius function

diameter function (width)

\[
\begin{align*}
0 & \rightarrow 2\pi \\
S & \rightarrow \mathbb{R} \\
S' & \rightarrow \mathbb{R}^2
\end{align*}
\]

Squeeze function \( S, S' \rightarrow S' \)

Piecewise constant monotone step function

\( \Theta_x \) where \( \Theta_x \) is leftmost point

Symmetry in object \( \rightarrow \) periodicity \( \rightarrow \) aliasing