Thermodynamic energy equation
(Ch. 2.6)

The first law of thermodynamics

internal energy change
+ work done by the air parcel
= external energy input

$$c_v \frac{DT}{Dt} + p \frac{D\alpha}{Dt} = J \qquad \alpha = \frac{1}{\rho}$$

Ideal gas law

$$p = \rho R T$$

$$p\alpha = RT$$

take total differential w.r.t. time

$$\boxed{} = R\frac{DT}{Dt}$$

Rewrite the first law of thermodynamics

$$\boxed{} = J$$

divide by T

$$\boxed{} = \frac{J}{T} \equiv \frac{Ds}{Dt}$$

Thermodynamic energy equation
(Ch. 2.6)

The first law of thermodynamics with J=0 (adiabatic)

in differential form

$$\boxed{} = 0$$

Potential temperature (Θ)

integrate above from p, T to p_s, Θ

$$\theta = \boxed{}$$

take logarithm and total differential w.r.t. time

$$c_p \frac{D \ln \theta}{Dt} = \boxed{}$$

Rewrite the first law of thermodynamics

$$\boxed{} = \frac{J}{T}$$

Scale analysis of the thermodynamic energy equation (Ch. 2.7.4)

Thermodynamic energy equation

$$\boxed{} = \frac{J}{T}$$

Decomposition of potential temperature

$$\theta(x, y, z, t) = \theta_0(z) + \theta'(x, y, z, t)$$

$$\frac{1}{\theta}\frac{D\theta}{Dt} = \frac{1}{(\theta_0 + \theta')}\frac{D(\theta_0 + \theta')}{Dt}$$

$$\approx \frac{1}{\theta_0}\left(1 - \frac{\theta'}{\theta_0}\right)\frac{D(\theta_0 + \theta')}{Dt}$$

$$\approx \boxed{}$$

$$\approx \boxed{}$$

Scale analysis of the thermodynamic energy equation
(Ch. 2.7.4)

Scale analysis of the thermodynamic energy equation

after decomposition

$$\boxed{} + c_p \frac{w}{\theta_0}\frac{d\theta_0}{dz} = \frac{J}{T}$$

$c_p = 1004\, J kg^{-1} K^{-1}$

$|u, v| \sim 10\; m\; s^{-1}$

$|w| \sim 10^{-2}\; m\; s^{-1}$

$|\delta x, \delta y| \sim 10^{6}\; m$

$|\delta z| \sim 10^{4}\; m$

$|\delta t| \sim |\delta x / u| \sim 10^{5}\; s$

$|\delta\theta' / \theta_0| \sim 10^{-2}$

$|(\delta\theta_0)_z / \theta_0| \sim 10^{-1}$

$|J| < 10^{-2}\; W\; kg^{-1}$

Continuity equation scaled for midlatitude synoptic scale motions

$$\boxed{} = 0$$

where are radiation, condensation, sensible heat flux, etc.?

Adiabatic lapse rate and static stability
(Ch. 2.7.2, 2.7.3)

Adiabatic lapse rate

Start with the definition of potential temperature, take logarithm and total differential w.r.t. height

$$c_p \frac{\partial \ln \theta}{\partial z} = c_p \frac{\partial \ln T}{\partial z} - R \frac{\partial \ln p}{\partial z}$$

$$\frac{T}{\theta}\frac{\partial \theta}{\partial z} = \boxed{} = \boxed{}$$

If potential temperature is constant in vertical

$$\Gamma_d \equiv -\frac{\partial T}{\partial z} = \boxed{}$$

Static stability

If potential temperature varies in vertical

$$\Gamma \equiv -\frac{\partial T}{\partial z}$$

$$\frac{T}{\theta}\frac{\partial \theta}{\partial z} = \boxed{}$$

stratification?

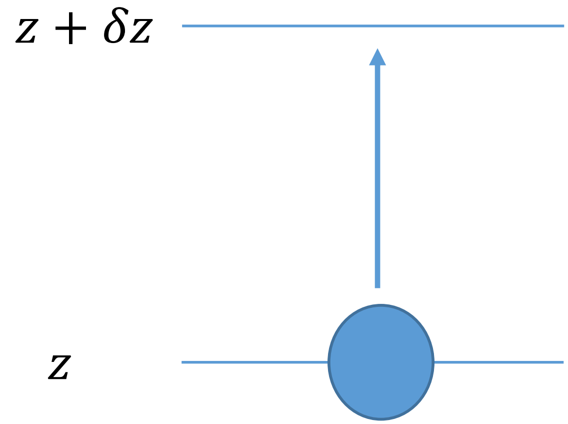
$\frac{T}{\theta}\frac{\partial \theta}{\partial z} > 0$ $\quad \Gamma_d \quad \Gamma$

$\frac{T}{\theta}\frac{\partial \theta}{\partial z} = 0$ $\quad \Gamma_d \quad \Gamma$

$\frac{T}{\theta}\frac{\partial \theta}{\partial z} < 0$ $\quad \Gamma_d \quad \Gamma$

Adiabatic lapse rate and static stability
(Ch. 2.7.2, 2.7.3)

What would be the vertical acceleration that the air parcel would get when it is adiabatically moved up by delta z?



$$\rho_0(z + \delta z) = \rho_0(z) + \boxed{\qquad}$$

$$\theta_0(z + \delta z) = \theta_0(z) + \boxed{\qquad}$$

$$\rho_0(z)$$

$$\theta_0(z)$$

(unscaled) vertical momentum equation after decomposing pressure and density into resting atmosphere and perturbation

$$\frac{Dw}{Dt} - \frac{u^2 + v^2}{a} = \boxed{\qquad\qquad} - g - 2\Omega\, u \cos \emptyset + \nu \nabla^2 w$$

the parcel has no horizontal motion, assume pressure adjust to environmental value quickly

$$\frac{Dw}{Dt} = \boxed{\qquad\qquad\qquad}$$

Adiabatic lapse rate and static stability
(Ch. 2.7.2, 2.7.3)

the acceleration of the air parcel

$$\frac{\mathrm{Dw}}{\mathrm{Dt}} = \frac{\mathrm{D}^2 \delta z}{\mathrm{Dt}^2} = \boxed{}$$

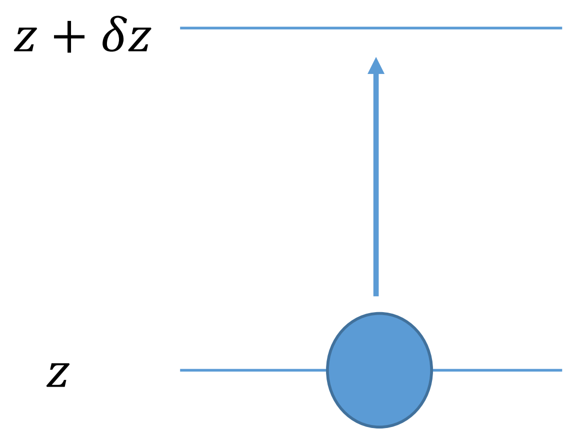
$$\frac{\mathrm{D}^2 \delta z}{\mathrm{Dt}^2} = -N^2 \delta z$$

Buoyancy frequency

$$N^2 = \boxed{}$$

general solution

$$\delta z = \boxed{}$$



What would be the trajectory of the air parcel if $N^2>0$?

What's the relationship between the buoyancy frequency and the static stability?