# Equations of motion

(Coriolis, centrifugal, pressure gradient, gravitational, and viscous forces)

- Equation that describes time changes in momentum *in a fixed frame of reference*

$$\frac{D_a \overrightarrow{U_a}}{Dt} = -\frac{1}{\rho}\nabla \cdot p - \overrightarrow{g^*} + \overrightarrow{F_r}$$

- We want it to describe atmospheric motion *relative to the Earth*

$$\frac{D\vec{U}}{Dt} + 2\,\vec{\Omega} \times \vec{U} - \Omega^2\vec{R} = -\frac{1}{\rho}\nabla \cdot p - \overrightarrow{g^*} + \overrightarrow{F_r}$$

- Rearrange (*the centrifugal force + true gravitational force = apparent gravitational force)

$$\frac{D\vec{U}}{Dt} = -\frac{1}{\rho}\nabla \cdot p - \overrightarrow{g^*} + \Omega^2\vec{R} - 2\,\vec{\Omega} \times \vec{U} + \overrightarrow{F_r}$$

- Equations of motion we will use

$$\frac{D\vec{U}}{Dt} = -\frac{1}{\rho}\nabla \cdot p - \vec{g} - 2\,\vec{\Omega} \times \vec{U} + \overrightarrow{F_r}$$

Equations of motion – in the component form
(Coriolis, pressure gradient, gravitational, and viscous forces)

- Zonal

$$\frac{\mathrm{Du}}{\mathrm{Dt}} - \frac{uv\tan\emptyset}{a} + \frac{uw}{a} = -\frac{1}{\rho}\frac{\partial \mathrm{p}}{\partial \mathrm{x}} + 2\Omega\,\mathrm{v}\sin\emptyset - 2\Omega\,\mathrm{w}\cos\emptyset + \nu\nabla^2\mathrm{u}$$
$$\frac{\mathrm{Du}}{\mathrm{Dt}} = -\frac{1}{\rho}\frac{\partial \mathrm{p}}{\partial \mathrm{x}} + 2\Omega\,\mathrm{v}\sin\emptyset$$

- Meridional

$$\frac{\mathrm{Dv}}{\mathrm{Dt}} - \frac{u^2\tan\emptyset}{a} + \frac{vw}{a} = -\frac{1}{\rho}\frac{\partial \mathrm{p}}{\partial \mathrm{y}} - 2\Omega\,\mathrm{u}\sin\emptyset + \nu\nabla^2\mathrm{v}$$
$$\frac{\mathrm{Dv}}{\mathrm{Dt}} = -\frac{1}{\rho}\frac{\partial \mathrm{p}}{\partial \mathrm{y}} - 2\Omega\,\mathrm{u}\sin\emptyset$$

- Vertical

$$\frac{\mathrm{Dw}}{\mathrm{Dt}} - \frac{u^2 + \mathrm{v}^2}{a} = -\frac{1}{\rho}\frac{\partial \mathrm{p}}{\partial \mathrm{z}} - g - 2\Omega\,\mathrm{u}\sin\emptyset + \nu\nabla^2\mathrm{w}$$
$$0 = -\frac{1}{\rho}\frac{\partial \mathrm{p}}{\partial \mathrm{z}} - g$$