AMATH 352: Problem set 6

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Question:	1	2	3	4	5	Total
Points:	12	9	2	8	9	40

1. **Geometric Transformations** I have supplied you with a "smiley.m" file that defines an "smiley" using (x,y) coordinates. Define matrices to preform the following operations on A. Apply the transformation and include a plot of each transformed output in your write up. I have supplied an example where I applied a shear matrix plot the output.
   (a) (3 points) Rotate by 45° counter-clockwise. (Consider: $x_{new} = ay + bx$, and $y_{new} = cx + dy$ that result in rotating the line) Save the matrix as 'A1aM.dat', and the resulting data as 'A1aArot.dat' in your scorelator script.
   (b) (3 points) Scale in the x direction by 2. Save the matrix as 'A1bM.dat', and the resulting data as 'A1bAscale.dat' in your scorelator script.
   (c) (3 points) Reflect across the line y = x. Save the matrix as 'A1cM.dat', and the resulting data as 'A1cAreflect.dat' in your scorelator script.
   (d) (3 points) Shear in the x direction by a factor $k = 2$ ($x_{new} = y + kx$, and $y_{new} = y$). Save the matrix as 'A1dM.dat', and the resulting data as 'A1dAshear.dat' in your scorelator script.

2. **Calculating eigenvalues and vectors by hand:**

   Compute the eigenvalues and eigenvectors of the following matrices using the characteristic equation. Show your work in your written solution.
   (a) (3 points) $\mathbf{B}_1 = \begin{bmatrix} 2 & 7 \\ 7 & 2 \end{bmatrix}$
   (b) (3 points) $\mathbf{B}_2 = \begin{bmatrix} 2 & 6 \\ 2 & 1 \end{bmatrix}$
   (c) (3 points) $\mathbf{B}_3 = \begin{bmatrix} 1 & -1 \\ 4 & 1 \end{bmatrix}$

3. **Eigenvalue/eigenvector decomposition as geometric transformation** Calculate the eigenvalue/eigenvector decomposition for 2a, $\mathbf{B}_1 = \mathbf{R}\boldsymbol{\Lambda}\mathbf{R}^T$ using Matlab `[R,L] = eig(B1)`. Note, that because only the direction the eigenvectors points matters, you may not have gotten the same magnitude eigenvectors in your by-hand calculation. This is fine.
   (a) (1 point) Apply the eigenvector matrix, R, to the data from smiley.m. Save the matrix, R, as 'A3aR.dat', and the resulting transformed data as 'A3aARtrans.dat' in your scorelator script.
   (b) (1 point) Apply the eigenvalue matrix, L, to the data from smiley.m. Save the L matrix as 'A3bL.dat', and the resulting transformed data as 'A3bLAtrans.dat' in your scorelator script.

   Quiz 6: What functional effect do the eigenvector and eigenvalue decomposition matrices have when applied to the smiley file? You do not need to save and submit the figures, but you might want to look at them.

4. **Matrix behavior for $\mathbf{A}^n$ as $n \to \infty$** Calculate the sequence $\mathbf{A}$, $\mathbf{A^2}$, $\mathbf{A^3}$, ... $\mathbf{A^n}$ for each of the following matrices in (a-c).

   (a) (2 points) $\mathbf{A_1} = \begin{bmatrix} 0.559 & 0.6 & 0.1 \\ 0.7 & 0 & 0 \\ 0 & 0.3 & 0 \end{bmatrix}$. Save the iterations for $n = 10, 100, 1000$ as 'A4an10.dat', 'A4an100.dat', and 'A4an1000.dat' in your scorelator script.

   (b) (2 points) $\mathbf{A_2} = \begin{bmatrix} 0.21 & 0.64 & 0.12 \\ 0.69 & 0 & 0 \\ 0 & 0.36 & 0 \end{bmatrix}$. Save the iterations for $n = 10, 100, 1000$ as 'A4bn10.dat', 'A4bn100.dat', and 'A4bn1000.dat' in your scorelator script.

   (c) (2 points) $\mathbf{A_3} = \begin{bmatrix} 1 & 0.4 & 0.2 \\ 0.3 & 0 & 0 \\ 0 & 0.2 & 1 \end{bmatrix}$. Save the iterations for $n = 10, 100, 1000$ as 'A4cn10.dat', 'A4cn100.dat', and 'A4cn1000.dat' in your scorelator script.

   (d) (2 points) For the transition matrix $\mathbf{A}_4 = \begin{bmatrix} 0.5 & 0.44 & 0.06 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ and initial population vector $\mathbf{x}_0 = \begin{bmatrix} 14 & 20 & 11 \end{bmatrix}$. Use matrix multiplication to see how the population changes over time. Save the population $\mathbf{x}_n$ at $n = 10, 100, 1000$ as 'A4dxn10.dat', 'A4dxn100.dat', and 'A4dxn1000.dat' in your scorelator script. Note, you are saving the population vector x_n, not the matrix A^n.

   Quiz 6: Determine if each sequence is stable, neutral stable or unstable for each population matrix **A**.

5. **Calculating largest eigenvalues**

   (a) Write an iterative algorithm using the power method with Rayleigh Quotient to find the largest eigenvalue of a general matrix **A**. Use a vector of all ones as your initial guess. Stop when the eigenvalue is within a tolerance of $tol = 10^{-4}$.

   Make sure you follow this order/schematic in your code– as shown in the YouTube video:

   - initialize placeholder eigenvector old and new (to use in the tolerance check)
   - start the while loop
   - save your old eigenvalue guess
   - update your eigenvector v= A*x
   - update your eigenvalue using Rayleigh
   - normalize the eigenvector guess.

   (b) Use your algorithm to calculate the largest eigenvalue and eigenvector for the matrices defined in question 4. You can confirm your results using MATLAB command [s d]=eig(A). The values will be slightly different, because of the error tolerance. Use the `format long` command in MATLAB to see more digits in the MATLAB output.

      i. (3 points) $\mathbf{A_1}$ Save the eigenvalue as 'A5bilambda.dat' in your scorelator script.
      ii. (3 points) $\mathbf{A_2}$ Save the eigenvalue as 'A5biilambda.dat' in your scorelator script.
      iii. (3 points) $\mathbf{A_3}$ Save the eigenvalue as 'A5biiilambda.dat' in your scorelator script.

   Quiz 6: How does the largest eigenvalue relate to your conclusions about stability in question 4?

Scorelator submission summary:

- 'A1aM.dat' and 'A1aArot.dat'
- 'A1bM.dat' and 'A1bAscale.dat'
- 'A1cM.dat' and 'A1cAreflect.dat'
- 'A1dM.dat' and 'A1dAshear.dat'

- 'A3aR.dat' and 'A3aARtrans.dat'
- 'A3bL.dat' and 'A3bLAtrans.dat'
- 'A4an10.dat', 'A4an100.dat', and 'A4an1000.dat'
- 'A4bn10.dat', 'A4bn100.dat', and 'A4bn1000.dat'
- 'A4cn10.dat', 'A4cn100.dat', and 'A4cn1000.dat'
- 'A4dxn10.dat', 'A4dxn100.dat', and 'A4dxn1000.dat'
- 'A5bilambda.dat', 'A5biilamdba.dat', and 'A5biiilambda.dat'

Written submission:

- Figure of original smiley and 45^o degree rotation
- Figure of original smiley and scaling x2 in the x-direction
- Figure of original smiley and reflection across the line x = y
- Figure of original smiley and shear in the x direction by a factor of 2.
- Problem 2 (a-c)