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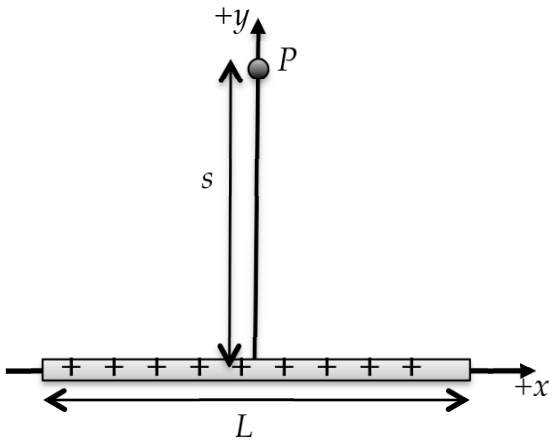
Question I. Line of Charge (27 pts total)

Consider a total charge $Q > 0$ spread uniformly over a thin rod of length L as shown. The rod lies along the x-axis and is centered at the origin. Point P is on the + y-axis a distance s above the origin.



1. **[3 pts]** What is the direction of the electric field at point P?

A. +**x** B. – **x** C. +**y** D. –**y** E. some other direction (or not enough information)

2. **[4 pts]** In the limit that $s >> L$ (point P is far away from the rod), which of these formulas is the best estimate for the magnitude of the electric field at point P (k=1/4πε$_o$)?

A. $E_P \sim \frac{kQ}{s^2}$ B. $E_P \sim \frac{kQ}{s}$ C. $E_P \sim \frac{kQ^2}{s^2+L^2}$ D. $E_P \sim \frac{kQ}{sL}$ E. $E_P \sim \frac{2kQ}{sL}$

3. **[4 pts]** In the limit that $s << L$ (point P is very close to the rod), which of these formulas is the best estimate for the magnitude of the electric field at point P?

A. $E_P \sim \frac{kQ}{s^2}$ B. $E_P \sim \frac{kQ}{s}$ C. $E_P \sim \frac{kQ^2}{s^2+L^2}$ D. $E_P \sim \frac{kQ}{sL}$ E. $E_P \sim \frac{2kQ}{sL}$

4. **[4 pts]** Which of these expressions best represents the electric field at point P in the general case?

A. $E_P = kQ\int_{-L/2}^{L/2}\frac{dx}{x^2+s^2}$ B. $E_P = \frac{kQ}{L}\int_{-L/2}^{+L/2}\frac{sdx}{x^2+s^2}$ C. $E_P = \frac{kQ}{L}\int_{-L/2}^{+L/2}\frac{xdx}{x^2+s^2}$

D. $E_P = \frac{kQ}{L}\int_{-L/2}^{+L/2}\frac{sdx}{(x^2+s^2)^{3/2}}$ E. $E_P = \frac{kQ}{L}\int_{-L/2}^{+L/2}\frac{xdx}{(x^2+s^2)^{3/2}}$

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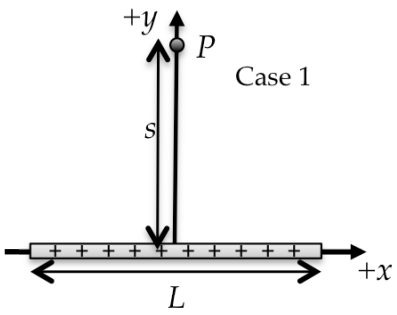
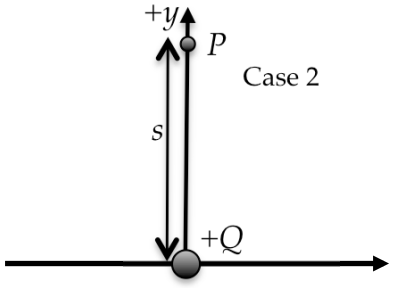




In Case 2, the charged rod of Case 1 is replaced by a point charge of the same total magnitude Q, located at the origin.

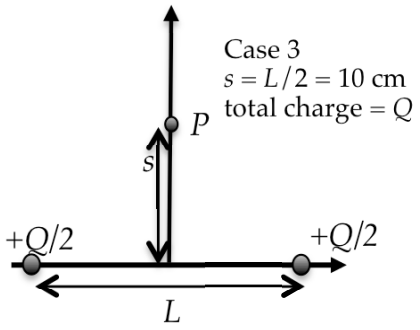
5. **[4 pts]** Which of the following statements best describes the relationship between the magnitude of the electric field at point P in Case 1 (rod) and in Case 2 (point charge)?

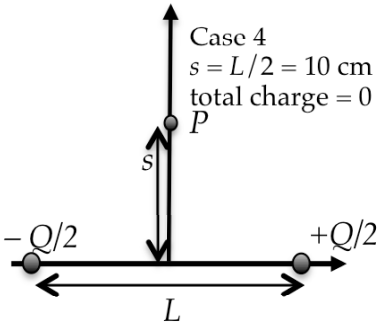
A. $E_1(P) > E_2(P)$ B. $E_1(P) = E_2(P)$ C. $E_1(P) < E_2(P)$ D. not enough information

Let s = 10.0 cm, L = 20.0 cm and $Q = 3.0 \times 10^{-9}$ C.

6. **[4 pts]** For Case 2 (point charge), what is the magnitude of the electric field at point P?

A. 0.27 N/C
B. 3.0×10^{-7} N/C
C. $2.7 \times 10^{+3}$ N/C
D. 8.1×10^{-6} N/C
E. none of these.





In Case 3, the same total charge Q is split into two point charges of magnitude $Q/2$, located on the x-axis at $\pm L/2$. In Case 4, the geometry is the same, but the sign of the charge at $x = -L/2$ is negative. In both cases, $s = L/2 = 10$ cm, and $Q = 3$ nC.

7. **[4 pts]** Which of the following statements is true about the electric field at point P in Cases 3 and 4?

A. The magnitude of the field is equal in Case 3 and Case 4, but the direction is different.
B. Both the magnitude and direction of the field are the same in Case 3 and Case 4.
C. The field is zero for Case 3, and non-zero for Case 4.
D. The field is zero for Case 4, and non-zero for Case 3.
E. None of the above statements are true.

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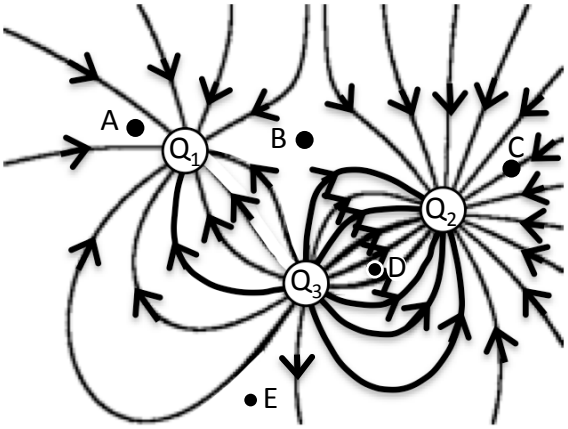
II. Fields and Charges [28 points total]
The picture at the right shows the electric field lines in a plane containing three point charges, Q_1, Q_2 and Q_3. Five locations (A through E) are marked on the diagram.



8. **[4 pts]** Which charges are positive?
   A. Only Q_1
   B. Only Q_2
   C. Only Q_3
   D. Both Q_1 and Q_2
   E. Not enough information

9. **[4 pts]** Which charge has the largest absolute magnitude?
   A. Q_1
   B. Q_2
   C. Q_3
   D. $|Q_2| = |Q_3| > |Q_1|$
   E. Not enough information

10. **[4 pts]** At which of the marked locations (A through E) does the electric field have the largest magnitude?

    A. A B. B C. C D. D E. E

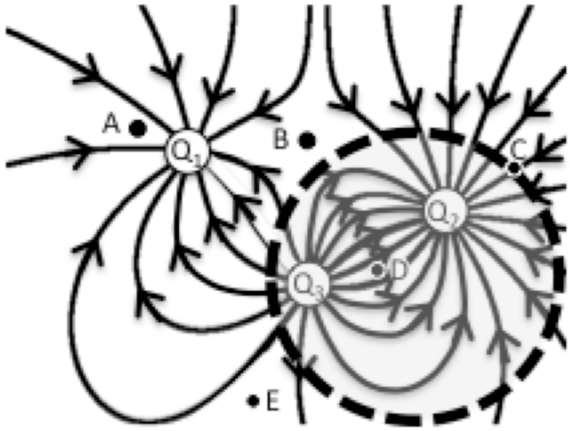
11. **[4 pts]** At which of the marked locations (A through E) is the electric field closest to zero magnitude?

    A. A B. B C. C D. D E. E

12. **[4 pts]** Which of these vectors best describes the direction of the *net force on an electron* located at position A?

A. ↖ B. ← C. ↘ D. ↑ E. no net force

The shaded area bounded by a dashed line depicts the cross section of a spherical Gaussian surface of radius R surrounding part of the same charge distribution as above. Point C is on the surface.



13. **[4 pts]** What is the sign of the net flux Φ_e through the Gaussian surface?
   A. Positive
   B. Negative
   C. Zero
   D. Not enough information

14. **[4 pts]** Let E_C = the magnitude of the electric field at point C. Which of the following statements is true?

A. $E_C > \dfrac{|\Phi_e|}{4\pi R^2}$ B. $E_C = \dfrac{|\Phi_e|}{4\pi R^2}$ C. $E_C < \dfrac{|\Phi_e|}{4\pi R^2}$

D. $E_c = \dfrac{|Q_2 + Q_3|}{4\pi\varepsilon_o R^2}$ E. $E_c = \dfrac{|Q_2|}{4\pi\varepsilon_o R^2}$

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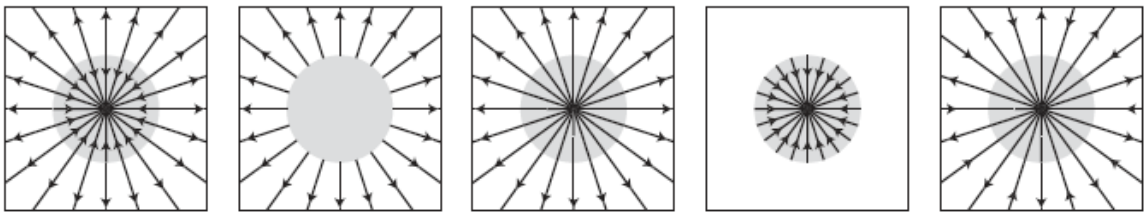
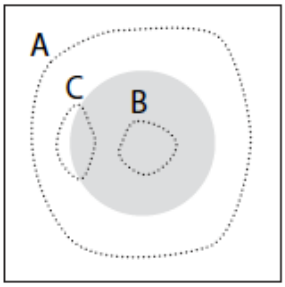
Question III. Mystery Charged Object [25 points total]
UW Astronomer Pablo Wiggins has discovered an extraordinary object and you have been hired to help his students interpret their astronomical observations. By studying light emitted from interstellar gas, the astronomers claim to have measured an earth-sized object with the following electric field, where R_o is the radius of the object and $E_o > 0$: $E(r) = \begin{cases} E_o\left(\frac{R_o}{r}\right)^2 \hat{\mathbf{r}} \; for \; r > R_o \\ 0 \qquad\qquad for \; r < R_o \end{cases}$.

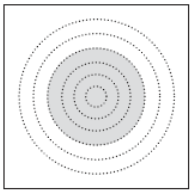
15. **[6 pts]** Pablo has sketched a few models for the electric field lines generated by the mystery object. Circle the correct answer, and explain your reasoning.

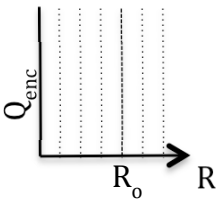


16. **[7 pts]** Sketch your model for the field lines on this diagram, and use the technique of counting field lines to estimate the relative flux through each surface. What do your results for flux imply about the location of the charge on the mystery object. Explain your reasoning.



17. **[7 pts]** Compute the charge inside successively larger concentric spherical shells as a function of R_o and E_o, and plot your result as a function of the radius of the shell. Explain the meaning of this calculation and its physical significance.





18. **[5 pts]** What physical properties (conducting, insulating, solid, hollow, etc.) of the mystery object could lead to your above results. Explain your reasoning.

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Question IV. Rods and Pith Balls [20 points total]
The following question is composed of two independent parts, A and B.
A. You are given two rods, labeled A and B, a small metal ball suspended on an insulating string, and some wool cloth. One of the rods is made of acrylic, which becomes positively charged when rubbed by wool. The other is made of Teflon, which becomes negatively charged when rubbed by wool. You do not know which rod is made of which material.

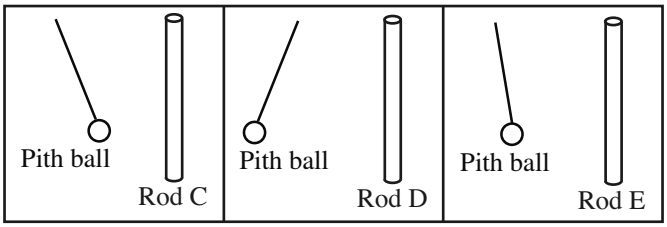
You rub the rods with the wool, and then hold each one next to the metal ball, one at a time.

19. **[5 pts]** Suppose the metal ball is uncharged. How will it react to the acrylic rod? To the Teflon rod? Explain.

20. **[5 pts]** Now suppose the metal ball is charged, but you don't know the sign of the charge on it. Could you determine which rod is acrylic and which is Teflon? Explain.

B. You obtain 3 new rods, labeled C, D, and E. One rod is acrylic, one rod is Teflon, and one rod is aluminum (a conductor). It is observed that the aluminum rod does not acquire a charge when rubbed by wool.

21. **[4 pts]** You rub the rods with wool, and bring each one close to the metal ball in turn (see figure). The ball does not touch any of the rods and no sparks occur.
The metal ball is attracted to C, repelled from D, and is slightly attracted to E. Based on these interactions, can you tell if the metal ball is charged? Explain.



22. **[6 pts]** Based on these interactions, what can you deduce about which rod is made from which material? Can you completely determine the material from which each rod is made? Explain.