10/12/2016  Lecture 7 Announcements:

- Problem Set 1 Due Today. Quiz 1 Due Tomorrow, Thursday 10/13.
- Discussion board participation due Friday 10/14.
- Problem Set 2 will be posted by 5pm today.

Last Time:

- Does the solution exist? Singular, and nonsingular matrices
- Calculating Inverses
- LU decomposition
- Determinants

Today:

- Re-formulating Inverse and LU decomposition in terms of equations
- Permutation Matrices and an easy trick for calculating LU
- Operation Counting

Calculating the inverse using Gaussian Elimination is the same as solving AX=I

recall $B = A^{-1}$ if $AB = I$

or solving $Ax = b$ n times for nxn matrix.

$$\begin{bmatrix} A \end{bmatrix}\begin{bmatrix} X \end{bmatrix} = \begin{bmatrix} 1 & & 0 \\ & 1 & \\ 0 & & 1 \end{bmatrix} \xrightarrow{GE} \begin{bmatrix} 1 & & 0 \\ & 1 & \\ 0 & & 1 \end{bmatrix}\begin{bmatrix} X \end{bmatrix} = \begin{bmatrix} A^{-1} \end{bmatrix}$$

$X$: unkown

LU decomposition is "pausing" half-way through (almost)

$$\left[\begin{array}{c|c} A & \begin{matrix} 1 & & 0 \\ & 1 & \\ 0 & & 1 \end{matrix} \end{array}\right] \Rightarrow \left[\begin{array}{c|c} \begin{matrix} 1 & & 0 \\ \ell_{21} & \ddots & \\ \ell_{n1} & \cdots & \ell_{n-1,n}\ 1 \end{matrix} & \begin{matrix} 1\ u_{12} & \cdots & u_{1n} \\ & \ddots & \vdots \\ 0 & & u_{n-1,n} \\ & & 1 \end{matrix} \end{array}\right] \quad L^{-1}I$$

We can avoid having to calculate the inverse of $L^{-1}$ by "breaking up $L^{-1}$" into simple lower triangular matrices and using two characteristics of simple lower triangular matrices. We also need the fact that $(AB)^{-1} = A^{-1}B^{-1}$.

1)If a lower triangular matrix has only 1 column of non-zero entries below the diagonal $L_i = -L_i^{-1}$

2) Many of these simple or "unit" lower triangular matrixes can be easily multiplied together

So we can construct the L without using Gaussian Elimination to calculate L from $L^{-1}$ :

Operation Counting for Gaussian Elimination on an NxN matrix:

Going from counting to computational complexity: Big O() notation.