CHAPTER I

The Charm of Impossibilities and the Relation of the Different Subject Matters

Knowing that music is a language, we shall seek at first to make melody "speak." The melody is the point of departure. May it remain sovereign! And whatever may be the complexities of our rhythms and our harmonies, they shall not draw it along in their wake, but, on the contrary, shall obey it as faithful servants; the harmony especially shall always remain the "true, " which exists in a latent state in the melody, has always been the outcome of it. We shall not reject the old rules of harmony and of form; let us remember them constantly, whether to observe them, or to augment them, or to add to them some others still older (those of plainchant and Hindu rhythmicss) or more recent (those suggested by Debussy and all contemporary music). One point will attract our attention at the outset: the charm of impossibilities. It is a glistening music we seek, giving to the aural sense voluptuously refined pleasures. At the same time, this music should be able to express some noble sentiments (and especially the most noble of all, the religious sentiments exalted by the theology and the truths of our Catholic faith). This charm, at once voluptuous and contemplative, resides particularly in certain mathematical impossibilities of the modal and rhythmic domains. Modes which cannot be transposed beyond a certain number of transpositions, because one always falls again into the same notes; rhythms which cannot be used in retrograde, because in such a case one finds the same order of values again — these are two striking impossibilities. We shall study them at the end of Chapter V ("Nonretrogradable Rhythms") and in Chapter XVI ("Modes of Limited Transpositions"). Immediately one notices the analogy of these two impossibilities and how they complement one another, the rhythms realizing in the horizontal direction (retrogradation) what the modes realize in the vertical direction (transposition). After this first relation, there is another between values added to rhythms and notes added to chords (Chapter III: "Rhythms with Added Values"); Chapter XIII: "Harmony, Debussy, Added Notes"). Finally, we superpose our rhythms (Chapter VI: "Polyrhythm and Rhythmic Pedals"); we also superpose our modes (Chapter XIX: "Polymodality").
CHAPTER II

Rāgavardhana, Hindu Rhythm

Before continuing, I pause to specify that in my music, and in all the examples of this treatise, the values are always notated very exactly; hence, whether it is a question of barred passages or not, the reader and the performer have only to read and execute exactly the values marked. In the passages not barred, which are the most numerous, I have saved the use of the bar-line to mark the periods and to give an end to the effect of the accidentals (sharps, flats, etc.). If you desire more ample information, refer to Chapter VII: “Rhythmic Notations.”

1) Ametrical Music (1)

Maurice Emmanuel and Dom Mocquereau knew how to illuminate, the former, the variety of the rhythmic patterns of ancient Greece, the latter, that of the neumes of plainchant. That variety will instill in us already a marked predilection for the rhythms of prime numbers (five, seven, eleven, thirteen, etc.). Going further, we shall replace the notions of “measure” and “beat” by the feeling of a short value (the sixteenth-note, for example) and its free multiplications, which will lead us toward a music more or less "ametrical," necessitating precise rhythmic rules. Recalling that Igor Stravinsky, consciously or unconsciously, drew one of his most striking rhythmic procedures, the augmentation or diminution of one rhythm out of two:

diminution of A at the cross, B does not change) from the Hindu rhythm simhavikridita:

(A augments and diminishes progressively, B does not change), we shall in our turn address ourselves to Hindu rhythmic to infer from it our first rules.

2) Rāgavardhana

Cān padda, Hindu theorist of the thirteenth century, has left us a table of a hundred and twenty deci-tālas, or Hindu rhythms (2). We find in this table the rhythm rāgavardhana:

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(1) Translator’s note. — The phrase "ametrical music" is here used to mean a music with free, but precise, rhythmic patterns, in opposition to "measured" (i.e. equally barred) music.

(2) Translator’s note. — The table may be found in the Encyclopédie de la musique et dictionnaire du conservatoire, eds. Albert Lavignac and Lionello de la Laurencie (Paris: Delagrave, 1913-1931), Part I, Vol. I, pp. 301 ff. Rāgavardhana is number 93. Simhavikridita is number 34.
Let us reverse this rhythm:

Thus reversed, it contains three quarter-notes (A) and three eighth-notes (B), classic diminution of three quarter-notes; further, the dot added to the second eighth (at the cross), which renders the diminution inexact, opens to us a new perspective of augmentation or diminution (by addition or withdrawal of the dot) and, above all, constitutes an added value; finally, the fragment B is a non-retrogradable rhythm:

From these statements, very insignificant in appearance, we can conclude: first, it is possible to add to any rhythm whatsoever a small, brief value which transforms its metric balance; second, any rhythm can be followed by its augmentation or diminution according to forms more complex than the simple classic doublings; third, there are rhythms impossible to retrograde. Let us study all that in detail.
Technique de mon langage musical

OLIVIER MESSIAEN

II

Exemples musicaux

1. Strawinsky
   Sacre du Printemps,
   Danse sacrée.

2. A B A+ B
   simulati

3. vêgavardhana

4. A

5. B+

6. A B A B A B A B

7. flegato

8. B

9. flegato

10. Vif et joyeux
    les Anges
    Orgue
    OPB. Montres 8, 4, 2 et plein-jeu du R.
    flegato

11. Vif et joyeux
    les Anges
    Orgue
    flegato

12. Modéré, joyeux
    les bergers
    Clarinette
    Orque
    OPB. Montres 8, 4, 2 et plein-jeu du R.
    flegato

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CHAPTER IV

Augmented or Diminished Rhythms and a Table of These Rhythms

1) Augmented or Diminished Rhythms

J. S. Bach practiced the canon by augmentation or diminution; in it the values of the proposed theme are generally doubled or diminished by half. We ourselves shall have the statement of the rhythm followed by its immediate augmentation or diminution, and according to more or less complex forms. An example of simple augmentation:

\[ \text{example 20} \]

A has the value of five sixteenth-notes, B of five eighth-notes, C of five quarters; B is the augmentation of A, C its varied double augmentation. Augmentation by addition of the dot is much more interesting.

2) Addition and Withdrawal of the Dot

An example of augmentation by addition of the dot:

\[ \text{example 21} \]

Simple rhythm in A; the same in B, all notes dotted; at the cross, added value.

An example of diminution by withdrawal of the dot:

\[ \text{example 22} \]

Other forms of augmentation and diminution exist. We shall set up a table of them.

3) A Table of Some Forms of Augmentation or Diminution of a Rhythm

This table comprises: a) addition of a quarter of the values; b) addition of a third of the values; c) addition of the dot, or addition of half the values; d) classic augmentation, or addition of the values to themselves; e) addition of twice the values; f) addition of three times the values; g) addition of four times the values. And so much for augmentation.

Everything concerning diminution permits inverse examples, placed opposite the preceding, which are: a) withdrawal of a fifth of the values; b) withdrawal of a quarter of the values; c) withdrawal of the dot, or withdrawal of a third of the values; d) classic diminution, or withdrawal of half the values; e) withdrawal of two-thirds of the values; f) withdrawal of three-fourths of the values; g) withdrawal of four-fifths of the values.
This table, slightly abridged, figures in the Preface of my *Quatuor pour la fin du Temps*. I particularly used its rhythmic forms in "Joie et clarté des Corps glorieux" (Les Corps glorieux) and in the "Danse de la fureur, pour les sept trompettes" (Quatuor pour la fin du Temps). Excessive augmentations or diminutions would have drawn us into some very long or very short values, which would have rendered the examples hardly appreciable to hearing; we limit ourselves, therefore, to a few forms only, all based upon the same initial rhythm: long, short, long:

Each example situated at the left of the table presents first the normal rhythm, then its augmentation; each example situated at the right of the table presents first the normal rhythm, then its diminution.

4) Inexact Augmentations

We saw in Chapter II, article 1, the augmentation of one rhythm out of two, which already gave a presentiment of the present paragraph. Here now are some examples of more striking inexact augmentations. First example:

B is the inexact augmentation of A; normally, the f-sharp' would have to be a dotted quarter-note. Second example:

At the cross, added value; B is the augmentation of A, C is the augmentation of B; the normal augmentation would be:

Let us notice further in this passage the use of the six-four chord with added sixth and added augmented fourth (see Chapter XIII). With very inexact augmentations or diminutions, one arrives at making rhythmic variants rather than augmentations or diminutions properly so called.
CHAPTER V

Nonretrogradable Rhythms

1) Retrograde Rhythms

One knows that retrogradation is a contrapuntal procedure which consists of reading from right to left what normally ought to be read from left to right. Applied to rhythm alone, it gives some curious reversals of values. Let us suppose the rhythmic formula:

We shall find it again in Chapters VI, XV and XIX. A typical formula of our rhythmic fancies, it contains a combination of augmented rhythms and added values, and, at the same time, inexact augmentations and diminutions; further, it begins with an interpretation of the ragavardhana, already seen in Chapter II; finally, the total of its values is thirteen quarter-notes (a prime number). All the fragments B are in diminution or augmentation of the fragments A; the added values are indicated by the crosses. Let us retrograde our formula; the order of the values is completely reversed, the diminutions changing themselves into augmentations and vice versa:

We shall see in Chapter VI the superposition of a rhythm upon its retrograde.

2) Nonretrogradable Rhythms

I have already spoken of nonretrogradable rhythms in a rather clear manner in the Preface of my Quatuor pour la fin du Temps. Whether one reads them from right to left or from left to right, the order of their values remains the same. A very simple example:

Outer values identical, middle value free. All rhythms of three values thus disposed are nonretrogradable:

If we go beyond the figure of three values, the principle grows, and we should say: all rhythms divisible into two groups, one of which is the retrograde of the other, with a central common value, are nonretrogradable.
The group B is the retrograde of group A; the quarter tied to a sixteenth-note (central value whose duration equals that of five sixteenth-notes) is common to the two groups. A succession of nonretrogradable rhythms (one per measure):

The melodic movement:

is repeated and, from this fact, goes through some important rhythmic variants.

3) Relation of Nonretrogradable Rhythms and Modes of Limited Transpositions

I have already spoken, in the Preface of my Nativité du Seigneur, of my prized modes of limited transpositions. We shall study them very much at length in Chapters XVI, XVII, XVIII and XIX. Let us go back to the reflections of Chapter I and examine the relation which is established between these modes and nonretrogradable rhythms. These modes realize in the vertical direction (transposition) what nonretrogradable rhythms realize in the horizontal direction (retrogradation). In fact, these modes cannot be transposed beyond a certain number of transpositions without falling again into the same notes, enharmonically speaking; likewise, these rhythms cannot be read in a retrograde sense without one's finding again exactly the same order of values as in the right sense. These modes cannot be transposed because they are — without polytonality — in the modal atmosphere of several keys at once and contain in themselves small transpositions; these rhythms cannot be retrograded because they contain in themselves small retrogradations. These modes are divisible into symmetrical groups; these rhythms, also, with this difference: the symmetry of the rhythmic groups is a retrograde symmetry. Finally, the last note of each group of these modes is always common with the first of the following group; and the groups of these rhythms frame a central value common to each group. The analogy is now complete.

Let us think now of the hearer of our modal and rhythmic music; he will not have time at the concert to inspect the nontranspositions and the nonretrogradations, and, at that moment, these questions will not interest him further; to be charmed will be his only desire. And that is precisely what will happen; in spite of himself he will submit to the strange charm of impossibilities: a certain effect of tonal ubiquity in the nontransposition, a certain unity of movement (where beginning and end are confused because identical) in the nonretrogradation, all things which will lead him progressively to that sort of theological rainbow which the musical language, of which we seek edification and theory, attempts to be.
Minuit pile et face
Chant
Soprano

A
B

Agnau, Seigneur!

25

Arc-en-ciel
D’innocence

Chant
Orgue

B

Piano

B

Modéré

2 expressif

B

B

32

rythme

non rétrogradable

ppp (lointain)

Un peu vif

ppp legato

B.

(fonds et anches 16, 8, 4, mixtures)

ppp staccato

Orgue

Un peu vif

ppp

le Verbe

ppp

35

34

Dans de la fureur,

pour les sept trompettes.

33

Un peu vif

pp

A

B

36
6) Canon of Nonretrogradable Rhythms

Let us try a canon of nonretrogradable rhythms. Let us recall example 33 of Chapter V: succession of nonretrogradable rhythms (one per measure):

Here is that same succession in triple canon, gathered into a $\frac{2}{4}$:

Each nonretrogradable rhythm is bracketed.

7) Rhythmic Pedal

Rhythm which repeats itself indefatigably, in ostinato—I was saying in paragraph 3—without busying itself about the rhythms which surround it. The rhythmic pedal, then, can accompany a music of entirely different rhythm; or mingle with it as in example 310 of Chapter XV; again it can be superposed upon other rhythmic pedals (see example 43 of Chapter VI). Let us consider now the following fragment:

The clarinet sings the principal melody (do not forget that it sounds a tone lower than the notation). The light formulas of the violin create a secondary counterpoint. The harmonics with vibrato of the violoncello, which sound two octaves higher than the notation in round notes, are a first rhythmic pedal, whose airy sonority envelops and unifies all the rest in its mysterious halo; here is the rhythm of this pedal:

It is divided into two nonretrogradable rhythms, A and B, the second of these rhythms being composed of two groups, one of which is the retrograde of the other, with a central common value at the cross (this central value is in reality a half-note, coined in four eighth-notes, which changes nothing in the nonretrogradation). Of course, the rhythm is repeated several times consecutively in the course of the piece, thus constituting a rhythmic pedal; the quoted fragment contains in X, Y, Z, the first two expressions and the beginning of the third expression. This rhythmic pedal of fifteen values is at the same time a melodic pedal of five notes; there is, then, disproportion between the rhythm and the melody—in M, N, O, P, Q, R, S, T, eight expressions of the melodic pedal. (As the rhythmic pedal repeats a rhythm independent of the rhythms which surround it, melodic and harmonic pedals repeat melody and chord-succession independent of the melodies and chords which surround them; see article 1 of Chapter XV which treats pedal-groups.)
The piano in its turn executes a repeated succession of chords, forming at once a rhythmic pedal and a harmonic pedal. Here again, the number of chords, twenty-nine, is different from that of values, seventeen. The chords undergo thus some unexpected rhythmic variants. ("Rhythmicize your harmonies!" Paul Dukas used to say to his students.) Here is the rhythm of this second rhythmic pedal:

We recognize the succession of its values, already commented upon in Chapter V. In A, B, C, D, four expressions of the rhythmic pedal. In H and I, the first two repetitions of the twenty-nine chords.

Let us look anew at the chords of the piano: from the first to the second cross, they are "chords on the dominant" with appoggiaturas, according to the effect of the stained-glass window of Chapter XIV (article 1) — from the third to the fourth cross, they use the third mode of limited transpositions (see Chapter XVI) — from the fifth to the sixth cross, the second of these modes.

Let us notice also that the melodic pedal of the violoncello is written in the whole-tone scale, the use of which can be tolerated when it is thus mixed with harmonic combinations which are foreign to it.

Let us point out finally that the formulas divided by rests in the violin (a sort of pedal) are written in "bird style," as is the principal song of the clarinet (a model of this style). We shall speak of it again in Chapter IX ("Bird Song").
58
Danser de la fureur,
pour les sept trompettes.

59
Amen des anges, des saints, du chant des oiseaux.

60
Liturgie de cristal
Bien modéré, en poudrelement harmonieux

(comme un oiseau)

PP (son flûte)

PP (vibrato)

+ Pp legato (très enveloppé de pédales)

vers la pointe

Glissando

A.L. 20,227
CHAPTER IX

Bird Song

Paul Dukas used to say, “Listen to the birds. They are great masters.” I confess not having awaited this advice to admire, analyze, and notate some songs of birds. Through the mixture of their songs, birds make extremely refined jumbles of rhythmic pedals. Their melodic contours, those of merles especially, surpass the human imagination in fantasy. Since they use untampered intervals smaller than the semitone, and as it is ridiculous servilely to copy nature, we are going to give some examples of melodies of the “bird” genre which will be transcription, transformation, and interpretation of the volleys and trills of our little servants of immaterial joy.

A first example, drawn from my Quatuor pour la fin du Temps:

example 114

In A, an arpeggio on the dominant chord with appoggiaturas (Chapter XIV, article 1):

example 115

See too in Chapter VI, article 7, example 60 (“Liturgie de cristal”), also drawn from my Quatuor; read there the so fanciful melody of the clarinet, particularly typical of the bird style.

The call of a merle:

example 116

Four ornamental variations of a theme and its “commentary” (see Chapter XI, article 2) which were suggested to me by the improvisations of a merle:

example 117

The vehement tirralirra, always higher, of the lark:

example 118

Hymn of the sparrows at daybreak:

example 119
**113**
*Modéré, un peu lent, rêveur*
*Pos: clarinette et piano*

**114**
*Presque vif, gai, capricieux*

**115**
*p'expressif*

**116**

**117**
*Très modéré, avec fantaisie*

*A.L. 30,227*

* L'exemple 117 figure, avec quelques variantes, dans "Amen des anges, des saints, du chant des oiseaux".*
CHAPTER XVI

Modes of Limited Transpositions

I have already spoken of these modes in the Preface of my *Nativité du Seigneur*. Let us repeat that exposition, amplifying it considerably by examples and more detailed explanations of the mechanism of the modes. To lighten my text, I shall not constantly use the term, "modes of limited transpositions," which is a little long, but I shall designate each mode by its numeral: second mode, third mode, etc., or mode 2, mode 3, etc.

All the examples of this chapter use the chosen mode melodically and harmonically, that is to say, all their notes belong to the mode. In the contrary cases, I indicate the notes foreign to the mode.

1) Theory of the Modes of Limited Transpositions

Based on our present chromatic system, a tempered system of twelve sounds, these modes are formed of several symmetrical groups, the last note of each group always being common with the first of the following group. At the end of a certain number of chromatic transpositions which varies with each mode, they are no longer transposable, the fourth transposition giving exactly the same notes as the first, for example, the fifth giving exactly the same notes as the second, etc. (When I say "the same notes," I speak enharmonically and always according to our tempered system, C-sharp being equal to D-flat.) There are three modes of this type. There are four other modes, transposable six times, and presenting less interest, for the very reason of their too great number of transpositions. All the modes of limited transpositions can be used melodically, and especially harmonically, melody and harmonies never leaving the notes of the mode. We spoke in Chapter I of the charm of impossibilities; their impossibility of transposition makes their strange charm. They are at once in the atmosphere of several tonalities, *without polytonality*, the composer being free to give predominance to one of the tonalities or to leave the tonal impression unsettled. Their series is closed. It is mathematically impossible to find others of them, at least in our tempered system of twelve semitones. In the tempered system in quarter-tones, extolled by Haba and Wischnegradsky, there exists a corresponding series (unfortunately, I cannot busy myself here with it, no more
than with the other particularities of quarter-tone music, no more than with the
relations between tempered and untempered music, all questions which will
impassion the musicians of the future, but passing the boundaries of this work).
I add that the modes of limited transpositions have nothing in common with the
three great modal systems of India, China, and ancient Greece, no more than with
the modes of plainchant (relatives of the Greek modes), all these scales being
transposable twelve times.

2) First Mode of Limited Transpositions

The first mode is divided into six groups of two notes each; it is transposable
twice. It is the whole-tone scale. Claude Debussy, in Pelléas et Mélisande,
and after him Paul Dukas, in Ariane et Barbe-Bleue, have made such remark-
able use of it that there is nothing more to add. Then we shall carefully avoid
making use of it, unless it is concealed in a superposition of modes which renders
it unrecognizable, as in example 43 of Chapter VI, paragraph 3.

3) Second Mode of Limited Transpositions

One already finds traces of it in Sadko by Rimsky-Korsakov; Scriabine uses it
in a more conscious fashion; Ravel and Stravinsky have used it transiently.
But all that remains in the state of timid sketch, the modal effect being more or
less absorbed by classified sonorities.
Mode 2 is transposable three times, as is the chord of the diminished seventh.
It is divided into four symmetrical groups of three notes each. These " tri-
chords," taken in ascending movement, are themselves divided into two inter-
vals: a semitone and a tone. Here is the first transposition:

example 312

Here are the second and third transpositions:

examples 313 and 314

The fourth transposition gives exactly the same notes as the first (enharmoni-
cally speaking):

example 315

The fifth transposition gives the same notes as the second, the sixth the same as
the third, and so on. One can begin the scale on the second degree; we shall
thus have in each group the intervals of a tone, a semitone (instead of a semitone,
a tone, as previously); but that changes nothing in the chords created by the
mode, and we fall again, enharmonically, into the notes of the first transposi-
tion:

example 316

Mode 2, first transposition, in parallel succession of chords (each voice realizes
the entire mode, starting on a different degree):

example 317
This succession alternates the six-four chord with added augmented fourth and the dominant seventh chord with added sixth (see Chapter XIII).

Contrary motion, same transposition: example 318

Typical chord of the mode, same transposition: example 319

Chord containing all the notes of the mode, in its second transposition: example 320

Various formulas of cadence belonging to the second mode: examples 321 to 324

The first formula is the typical cadence of the mode, first transposition; we have already seen it in Chapter VIII (example 77, "la Vierge et l'Enfant"). The second formula uses the mode in its second transposition. The third formula is a progression of harmony; at A, first term, third transposition; at the cross, added value, giving more force to the preparation of the accent; at B, second term, first transposition, rhythmical variation; at the cross, value elongated by addition of the dot, slackening the descent. The fourth formula uses the second transposition.

I have already pointed out, in the examples of the preceding chapters, frequent borrowings from the second mode. New examples of its use: examples 325 to 327

The three examples do not leave the notes of the mode in its first transposition. The third example contains, in the piano, an interesting formula of accompaniment (see Chapter XIV, paragraph 8); compare its melodic movement with example 113 of Chapter VIII ("l'Ange aux parfums"). example 328

This last example uses mode 2: at A, in its third transposition; at B, in its first transposition.

4) Third Mode of Limited Transpositions

It is transposable four times, as is the chord of the augmented fifth. It is divided into three symmetrical groups of four notes each. These "tetrachords," taken in ascending movement, are divided themselves into three intervals: a tone and two semitones. Here is the first transposition: example 329

Here are the second, third, and fourth transpositions: examples 330 to 332

The fifth transposition gives the same notes as the first, the sixth the same notes as the second, and so on, according to the phenomenon observed in mode 2. One can begin the scale on the second or on the third degree, but (as we have seen also for the second mode) there follows only a new order of tones and semitones
for each group, no change of the notes constituting the mode or of the chords called for by it. Mode 3, first transposition, in parallel succession of chords (each voice realizes the entire mode starting on a different degree) :

Contrary motion, second transposition :

Typical chord, first transposition :

Chord containing all the notes of the mode, same transposition :

Two cadence formulas, the first in the fourth transposition :

the second, in contrary motion, in the first transposition :

Use of the third mode :

This fragment does not leave the notes of the mode in its first transposition. The cluster of chords, which is repeated from measure to measure in the upper staff of the piano, constitutes a pedal group (see Chapter XV). Another use of the third mode :

Use of the cadence formulas of examples 337 and 338, of the contrary motion of example 334. At A, fourth transposition; at B, first transposition; at C, second transposition. The three letters D indicate notes foreign to the mode, forming effects of superior and inferior resonances (see Chapter XIV, article 4).

5) Modes 4, 5, 6, and 7

These modes are transposable six times, like the interval of the augmented fourth. They are divided into two symmetrical groups. There are four of them, which carries the total number of modes of limited transpositions to seven. One cannot find others of them transposable six times, because all the other combinations dividing the octave into two symmetrical groups must : commence the scales of modes 4, 5, 6, and 7 upon other degrees than the first (which changes the order of the intervals, but not the notes or the chords of the modes — we have already established that); or form arpeggios of classified chords; or form truncated modes 2, such as :

or form truncated modes 5 :

Here is mode 4 :

The same in parallel succession of chords :
I have used this mode in "Prière exauisée" from my Poèmes pour Mi.
The same scale, less two notes; it is mode 5: example 347

This mode 5, being a truncated mode 4, has the right of quotation here only
because it engenders the melodic formula (already seen in Chapter X): example 348

and the chord in fourths, analyzed in Chapter XIV, paragraph 3: example 349

Both the chord in fourths and the melodic formula contain all the notes of
mode 5.

Now here is mode 6: example 350

Contrary movement in this mode: example 351

Another contrary movement: example 352

Same mode, parallel succession of chords over a sustained augmented fourth: example 353

For the use of mode 6, see in Chapter III, paragraph 2, example 12 ("les Berger")s, which uses the mode in its fifth transposition; and in Chapter VIII,
paragraph 4, example 109 ("la Vierge et l’Enfant"), which uses the mode in
its first transposition.

Finally, here is mode 7: example 354

The same, in parallel succession of chords: example 355

I have used this mode in the fourth part of l’Ascension: "Prière du Christ montant vers son Père." Let us recall example 253 of the list of chord connections
(Chapter XIV, paragraph 8): example 356

which uses all the notes of mode 7 in its fifth transposition: example 357

6) Relation of Modes of Limited Transpositions and Nonretrogradable
Rhythms

Let us return to the reflections of Chapter V, article 3. There it is said: "The
modes of limited transpositions realize in the vertical direction (transposition)
what nonretrogradable rhythms realize in the horizontal direction (retrogradation). In fact, these modes cannot be transposed beyond a certain number of
transpositions without falling again into the same notes, enharmonically speaking; likewise, these rhythms cannot be read in a retrograde sense without
one’s finding again exactly the same order of values as in the right sense. These
modes cannot be transposed because they are — without polytonality — in the
modal atmosphere of several keys at once and contain in themselves small transpositions; these rhythms cannot be retrograded because they contain in themselves small retrogradations. These modes are divisible into symmetrical groups; these rhythms, also, with this difference: the symmetry of the rhythmic groups is a retrograde symmetry. Finally, the last note of each group of these modes is always common with the first of the following group; and the groups of these rhythms frame a central value common to each group. The analogy is now complete."

Again in the same article 3 of Chapter V, it is said: "Let us think of the hearer of our modal and rhythmic music; he will not have time at the concert to inspect the nontranspositions and the nonretrogradations, and, at that moment, these questions will not interest him further; to be charmed will be his only desire. And that is precisely what will happen; in spite of himself he will submit to the strange charm of impossibilities: a certain effect of tonal ubiquity in the nontranspositions, a certain unity of movement (where beginning and end are confused because identical) in the nonretrogradation, all things which will lead him progressively to that sort of theological rainbow which the musical language, of which we seek edification and theory, attempts to be."

Arrived at this place in our treatise, is it not useful to repeat these lines?