INTRODUCTION

When Schoenberg composed the first twelve-tone piece in the summer of 1921,¹ the “Prelude” to what would eventually become his Suite, Op. 25 (1923), he carried to a conclusion the developments in chromaticism that had begun many decades earlier. The assault of chromaticism on the tonal system had led to the nonsystem of free atonality, and now Schoenberg had developed a “method [he insisted it was not a “system’] of composing with twelve tones that are related only with one another.”

Free atonality achieved some of its effect through the use of aggregates, as we have seen, and many atonal composers seemed to have been convinced that atonality could best be achieved through some sort of regular recycling of the twelve pitch classes. But it was Schoenberg who came up with the idea of arranging the twelve pitch classes into a particular series, or row, that would remain essentially constant throughout a composition.

Various twelve-tone melodies that predate 1921 are often cited as precursors of Schoenberg’s tone row, a famous example being the fugue theme from Richard Strauss’s Thus Spake Zarathustra (1895). A less famous example, but one closer than Strauss’s theme to Schoenberg’s method, is seen in Example 10–1. Notice that Ives holds off the last pitch class, C, for 3½ measures until its dramatic entrance in m. 68.

In the music of Strauss and Ives the twelve-note theme is a curiosity, but in the music of Schoenberg and his followers the twelve-note row is a basic shape that can be presented in four well-defined ways, thereby assuring a certain unity in the pitch domain of a composition.

This chapter presents the basics of “classical” serialism, the serial technique developed by Schoenberg and adopted by Webern and Berg (somewhat more freely by the latter),
as well as many other composers. Chapter 13 will deal with more advanced serial topics, concentrating on integral serialism.

EXAMPLE 10-1 Ives: Three-Page Sonata (1905), mm. 62–68  (© 1949 Mercury Music Corporation. Used by permission of the publisher.)

Allegro-March time

BASIC TERMINOLOGY

The core of the twelve-tone system is the tone row (basic set, series), an ordered arrangement of the twelve pitch classes (not twelve pitches), with each one occurring once and only once. The row itself has four basic forms:

1. **Prime**: the original set (not to be confused with the prime form of an unordered set, discussed in Chapter 9)
2. **Retrograde**: the original set in reverse order
3. **Inversion**: the mirror inversion of the original set
4. **Retrograde Inversion**: the inversion in reverse order

The row that Schoenberg used for his first serial work is shown in its four basic forms in Example 10–2. The notes could have been written here in any octave and with enharmonic spellings—it would still be the same row. We follow the convention in this and in similar examples of omitting natural signs; any note without an accidental is natural. The numbers under the notes are called order numbers and simply indicate each note’s position in the row form.
In addition, each of the four basic forms has twelve transpositions—that is, each one may be transposed to begin with any of the twelve pitch classes—so a single row has \(4 \times 12\), or 48, versions that are available to the composer. In simple terms, a twelve-tone work consists of the presentation of various row forms at various transpositions, though the details of how this is done vary from composer to composer and from piece to piece.

When analyzing a serial composition we label the row forms using abbreviations:

- \(P = \text{Prime}\)
- \(R = \text{Retrograde}\)
- \(I \neq = \text{Inversion}\)
- \(RI = \text{Retrograde Inversion}\)
After the abbreviation comes a number, from 0 to 11, which specifies the transposition level of the row. A prime form or an inversion that begins on C would have a transposition level of 0 (P-0 or I-0), one beginning on C♯/D♭ would have a transposition level of 1 (P-1 or I-1), and so on to the pitch class B which is represented by an 11. However, the transpositional level of an R or RI form is indicated by the pitch class that ends the row; R-0 and RI-0 would both end with a C because they are the retrogrades of P-0 and I-0. Therefore, the row forms in Example 10–2 are P-4, R-4, I-4, and RI-4.

THE TWELVE-TONE MATRIX

It is sometimes helpful when composing or analyzing serial music to be able to see all forty-eight versions of the row. The matrix, or “magic square,” allows you to see all forty-eight versions after writing out only twelve of them. Example 10–3 is the matrix for the row for Schoenberg’s Suite. The prime forms can be read from left to right along the rows of the matrix, while the retrogrades are read from right to left. The inversions are read along the columns from top to bottom, and the retrograde inversions from bottom to top. The transposition number is next to the first note of each row form. Looking down the left-hand side of the matrix, you can see that P-4 begins on E, P-3 on D♯, P-1 on C♯, and so on.

To fill in the matrix, follow these steps:

1. Write the prime form of the row along the top row of the matrix. It does not matter what transposition level you choose.
2. Fill in the main diagonal (the one that runs from upper left to lower right) with the first note in the top row of the matrix.
3. In the next row of the matrix, identify the interval between the note in the main diagonal and the note immediately above it.
4. Transpose the other eleven notes of that row by the same interval. Use simple spellings (not B♯ or F♯, for example), and make sure that there are exactly five notes with accidentals when you finish the row. (In Example 10–3, we have used all sharps, but you could use all flats or a combination of the two.)
5. Repeat Steps 3 and 4 until all twelve rows are filled.
6. Fill in the transposition levels along the top and left borders (only), with C = 0, C♯/D♭ = 1, and so on.
7. Copy the numbers from the left border onto the right border, and from the top border to the bottom one.
A FIRST EXAMPLE

Before going on to some more technical information, it would probably be of interest at this point to see how Schoenberg used the row we have been discussing. The beginning of the work is given in Example 10–4. Since this is the first serial piece that Schoenberg composed, you might expect it to be fairly simple in terms of row usage, but this is really not the case. While reading the discussion that follows the example, be sure to find in the matrix (Example 10–3) every row that is mentioned.

The Prelude is the first movement of the Suite, and the first row form to be used is P–4. Here the first row form occurs in the treble clef, beginning on E and ending on B♭. P–4 is
accompanied at the beginning by P-10, and the careful listener will hear the imitation between the two voices at this point:

\[
\begin{array}{cccc}
P-4: & E & F & G & D_b \\
P-10: & B_b & C_b & D_b & G \\
\end{array}
\]

P-10 continues toward the end of m. 2 in the tenor voice: C–A–D–G♯, in imitation of G♭–F♭–A♭–D in the soprano, while beneath the tenor the bass sounds the last four notes: F–F♯–E♭–E. Notice that notes 9–12 here do not follow notes 5–8, but occur simultaneously with them.

The B♭ that ends P-4 becomes the bass for a time, and it also serves as the first note of I-10, the next row form. Trace the first four notes of this row form, B♭–A–G–D♭, as they move from the bottom staff to the top staff and finally, in m. 5, to the melody. Some listeners would be able to recognize that the sixteenth-note line, B♭–A–G, is the inversion of the opening motive, E–F–G, and that G–D♭ occurs here in the same octave as in m. 1. The highest voice from the end of m. 3 through m. 4 is made up from notes 5–8, A♭–C–G♭–C, while the alto sounds notes 9–12, E♭–D–F–E.

To recapitulate: We have seen that P-4 and P-10 were used in counterpoint at the beginning, whereas in the next measures a single row form, I-10, accompanied itself. We have also seen that the row does not always have to proceed strictly from the first note to the last, but instead that segments of the row may appear simultaneously.

**EXAMPLE 10–4** Schoenberg: Suite, Op. 25 (1923), Prelude, mm. 1–5  (Used by permission of Belmont Music Publishers.)
ANALYZING A ROW

Since the tone row serves as the source of the pitch material of a composition, we really should analyze the row itself before beginning the analysis of the piece. The first step should be to play (or sing) it several times. Listen for sequences or familiar patterns. In general, composers avoid using in a row any combination of pitches that would recall tonal music, such as triads, scale segments, and traditional bass or melodic formulas. If the composer chooses to include such patterns, as occasionally happens, you should make note of this and its effect on the music. For example, play through the series Berg used for his Lyric Suite (1926):

\[
\begin{array}{cccccccccccc}
F & E & C & A & G & D & G\# & C\# & D\# & F\# & A\# & B \\
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12
\end{array}
\]

This row contains triads on A minor and D\# minor, and the row ends with a figure that suggests a B tonality, F\#–A\#–B. (The end of the retrograde, C–E–F, would suggest an F tonality.) The first hexachord (the first six notes) is diatonic to C major or F major, and the second hexachord is diatonic to F\# major or B major. Schoenberg's row (Example 10–2) contains fewer tonal references, but it ends with the retrograde of the famous B–A–C–H motive (in German B\# is written as B, and B\# as H), and we might expect Schoenberg to do something with this in the piece.

The next step in the analysis might be to label the ICs (interval classes; review Chapter 9) found between adjacent notes of the row. For instance, for Schoenberg's Op. 25 we find:

\[
\text{IC: } 1 \quad 2 \quad 6 \quad 5 \quad 3 \quad 5 \quad 6 \quad 3 \quad 1 \quad 3 \quad 1
\]

\[
\text{Note: } E \quad F \quad G \quad C\# \quad F\# \quad D\# \quad G\# \quad D \quad B \quad C \quad A \quad B\#
\]

\[
\text{Totals: } \text{IC1} \quad \text{IC2} \quad \text{IC3} \quad \text{IC4} \quad \text{IC5} \quad \text{IC6} \\
3 \quad 1 \quad 3 \quad 0 \quad 2 \quad 2
\]

We see from the totals (do not confuse this interval tabulation with the interval vector, discussed in Chapter 9) that there are no appearances of IC4 (major 3rd or minor 6th) and that IC1 (minor 3rd, major 7th) and IC3 (minor 3rd, major 6th) predominate. Some rows are composed so as to emphasize particular intervals, as is the case here, while others are not. The all-interval row, when spelled in an ascending fashion, contains exactly one appearance of each interval, from the minor 2nd through the major 7th. For example, the row from Berg's Lyric Suite:

\[
\begin{array}{cccccccccccc}
M7 & m6 & M6 & m7 & P5 & TT & P4 & M2 & m3 & M3 & m2 \\
F & E & C & A & G & D & G\# & C\# & D\# & F\# & A\# & B
\end{array}
\]

If the row that you are analyzing has two of each IC except for IC6, which appears once, check to see if it is an all-interval row.3

Some rows use the first three, four, or six notes as a pattern from which the rest of the row is derived: such a row is called a derived set. In such a set the pattern is transposed,
inverted, retrograded, or inverted and retrograded to "generate" the remainder of the set. For example, the first hexachord of Berg's row from the Lyric Suite in the previous paragraph in retrograde and transposed by a tritone would produce the second hexachord, so the row is a derived set. The row of Webern's Concerto, Op. 24 (1934), is generated by applying the RI, R, and I operations to the first trichord:

\[
\begin{array}{cccccccc}
B & B^b & D & E & G & F^\# & G^\# & E & F \\
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
pattern & RI of 1–3 & R of 1–3 & I of 1–3 \\
\end{array}
\]

Even if the row is not a derived set, it may well contain patterns that are transposed, inverted, or retrograded. Patterns of IC's that are repeated or reversed may help us to find these pitch-class patterns. In Schoenberg's Suite the repeated 3–1 at the end of the row is caused by two overlapping statements of a trichordal pattern:

IC: 1 2 6 5 3 5 6 3 1 3 1

Note: E F G C# F# D# G# D B C A B^b

\[
[D \ B \ C]
\]

[C A B^b]

The pattern 6–5 within the row is reversed later as 5–6, and it is also part of a larger palindromic pattern: 6–5–3–5–6. Such patterns may indicate that a segment of the row is its own retrograde or retrograde inversion. Here the patterns 6–5–3–5–6, 3–1–3 and 1–3–1 reproduce themselves under retrograde inversion. As it turns out, two of these segments are duplicated in RI-5, whereas the third is duplicated in RI-7, as shown in Example 10–5.

**EXAMPLE 10–5** Recurring row segments
These patterns that we have found may have implications in the piece motivically, and in this case they certainly have implications in the area of invariance. An invariant pitch class is one that is shared by any two collections of pitches (two chords, for example). Similarly, an invariant subset is one that appears intact in two forms of the row. The order of the notes was not important when dealing with unordered sets in Chapter 9, but it is important when talking about rows. Looking again at Example 10-5, you can see that RI-5 is a reorganized version of P-4, beginning with notes 9–12 from P-4, then notes 3–8, and ending with notes 1–2. Each prime row transposition in the matrix has a similar RI “cousin,” and there also are twelve similar IR pairs. The matrix of a derived set may exhibit even more invariance. The row from Berg’s Lyric Suite, for instance, has only twenty-four different row forms instead of the usual forty-eight, because each R form is the same as some P form, and each RI form is the same as some I form.

The next step in a thorough analysis is to analyze the subsets in terms of the pitch-class set types we learned in Chapter 9. This is not the same thing as the row segment analysis we were doing above, because there we were looking for exact transpositions (or inversions, etc.) of row segments. Here we are looking for set types, and we know that the notes of a pitch-class set may appear in any order and may be inverted to find the best normal order and the prime form. For example, there are ten trichords in any row, the first one made up of notes 1–3, the second of notes 2–4, and so on, with the last one made up of notes 10–12. There are also nine tetrachords, eight pentachords, and seven hexachords. The set types contained in Schoenberg’s row are the following:

<table>
<thead>
<tr>
<th>First Note</th>
<th>Trichords</th>
<th>Tetrachords</th>
<th>Pentachords</th>
<th>Hexachords</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>[0,1,3]</td>
<td>[0,2,3,6]</td>
<td>[0,1,2,3,6]</td>
<td>[0,1,2,3,4,6]</td>
</tr>
<tr>
<td>F</td>
<td>[0,2,6]</td>
<td>[0,1,2,6]</td>
<td>[0,1,2,4,6]</td>
<td>[0,1,2,3,5,7]</td>
</tr>
<tr>
<td>G</td>
<td>[0,1,6]</td>
<td>[0,1,4,6]</td>
<td>[0,1,2,5,7]</td>
<td>[0,1,2,5,6,7]</td>
</tr>
<tr>
<td>C#</td>
<td>[0,2,5]</td>
<td>[0,2,5,7]</td>
<td>[0,1,2,5,7]</td>
<td>[0,1,2,4,7,9]</td>
</tr>
<tr>
<td>F#</td>
<td>[0,2,5]</td>
<td>[0,1,4,6]</td>
<td>[0,1,4,6,9]</td>
<td>[0,1,3,4,7,9]</td>
</tr>
<tr>
<td>D#</td>
<td>[0,1,5]</td>
<td>[0,1,4,7]</td>
<td>[0,1,3,4,7]</td>
<td>[0,1,3,4,6,7]</td>
</tr>
<tr>
<td>G#</td>
<td>[0,3,6]</td>
<td>[0,2,3,6]</td>
<td>[0,1,3,4,6]</td>
<td>[0,1,2,3,4,6]</td>
</tr>
<tr>
<td>D</td>
<td>[0,1,3]</td>
<td>[0,2,3,5]</td>
<td>[0,1,2,3,5]</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>[0,1,3]</td>
<td>[0,1,2,3]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>[0,1,3]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>[0,1,3]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B♭</td>
<td>[0,1,3]</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In practice, it is usually enough to analyze only the trichords and tetrachords; the first and last hexachords will always be either of the same type or Z-related, and most of the pentachords and hexachords overlap too much to be of interest. The trichord analysis shows us that [0,1,3] and [0,1,6] are unifying elements in the set; the composer might possibly make use of them as melodic motives or as chords to help unify the piece. The set [0,2,5], although it occurs twice, is less useful because its two appearances overlap. The tetrachords [0,2,3,6] and [0,1,4,6] also might fulfill a unifying function, although the two appearances of [0,1,4,6] overlap by two notes.
stem up, and all three rows come together at this point. The use of I-8 also allows the first half of the piece to end in m. 6 on an inversionally symmetrical sonority, A-D-G♭-F, or (G347), which means that when the first half of the piece is inverted to form the second half, the final sonority will be a (transposed) duplication of this one.

The only remaining row choice to be discussed is the transposition level for the inversion of the melody, which begins with the C♭ in the bottom staff at the end of m. 6. The obvious answer is that I-11 is the only inversion that keeps the opening two dyads invariant:

\[
\begin{align*}
P-10 & : (A\# B) \quad (D\# F) \\
I-11 & : (B A\#) \quad (F D) \\
\end{align*}
\]

Another consideration might have been the nice G♭ major 7th sonority in m. 7 formed by the end of R-5 and the beginning of I-11.

**COMBINATORIALITY**

Sometimes the choice of row forms or transpositions is governed by a desire to form aggregates (without duplication of pitch class) between portions of row forms. For example, in the following diagram, the row that Schoenberg used for his Piano Piece, Op. 33a (1929), is followed by its RI-3 form. Notice that when the second hexachord of P-10 is combined with the first hexachord of RI-3, they form an aggregate. In effect, we have created a new row, called a secondary set, by combining two hexachords from two different row forms.

\[
\begin{array}{cccccccccccc}
B♭ & F & C & B & A \flat & C & D & G & A & D & E \\
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\
\hline
\end{array}
\begin{array}{cccccccccccc}
A & B & F & G♭ & B♭ & C & G & E & D & C & G & D \\
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\
\hline
\end{array}
\]

This combining of row forms to form aggregates is called **combinatoriality**, and it is an important aspect of some serial compositions. Most often, however, the combining is done vertically:

\[
\begin{array}{cccccccccccc}
\text{P-10:} & B♭ & F & C & B & A & D & G & A♭ & D & E \\
\text{I-3:} & D & G & C & D & E & G & C & B♭ & G♭ & F & B & A \\
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\
\hline
\end{array}
\]

This diagram is seen in notation in Example 10–8. The first aggregate occupies m. 14 through the first two notes of m. 16, and the second aggregate occupies the rest of the excerpt. Notice that Schoenberg freely retrogrades or repeats row segments, as in C–B–A–B–C in mm. 14–15.
Schoenberg’s row is so constructed that any pair of row forms that can be combined hexachordally to form twelve-tone aggregates can also be combined tetrachordally to form three sets of eight pitch classes each:

\[
\begin{array}{cccccccccccc}
\text{RI-3:} & A & B & F & F^\# & A^\# & C & G & E & D & C^\# & G^\# & D^\#
\end{array}
\]

\[
\begin{array}{cccccccccccc}
\text{R-10:} & E & D & A^b & G & D^\# & C^\# & F^\# & A & B & C & F & B^b
\end{array}
\]

\[
\begin{array}{cccccccccccc}
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12
\end{array}
\]

\[
\begin{array}{cccc}
\text{octachord} & \text{octachord} & \text{octachord}
\end{array}
\]

Though this does not produce twelve-tone aggregates in the way that the combined hexachords do, the technique is similar. In Example 10–9 each pair of tetrachords occupies approximately one measure.
Other rows are constructed to produce tetrachord aggregates by combining three rows vertically, or trichord aggregates by combining four rows vertically; however, hexachordal combinatoriality is the approach most commonly used.

Combinatoriality guarantees a more controlled recycling of the twelve pitch classes, and to some it seems a necessary extension of the twelve-tone aesthetic. Schoenberg invented this technique, although he obviously was not using it in his Suite (see the juxtaposed G/Db and D#/G in Example 10-4). Nor was Dallapiccola interested in combinatoriality in his Notebook (notice the duplicated G's in m. 3 of Example 10-7). In fact, most rows cannot by their nature be used combinatorially (except with their retrogrades) and must instead be specially constructed for that use. But combinatoriality has been of considerable interest to some composers, and a large number of pieces are combinatorial throughout.

**THE ANALYSIS OF SERIAL MUSIC**

In analyzing the use of rows in a serial piece, it is often enough to label the row forms (P, O, etc.) without writing the order numbers on the music. If the texture is complex or if some unusual row technique is being employed, it may be necessary to write the order numbers near the noteheads and even to join them with lines. Always work from a matrix. If you get lost, try to find several notes that you suspect occur in the same order in some row form, and scan the matrix for those notes, remembering to read it in all four directions.

It is important to understand that the labeling of row forms and the consideration of the details of their use is only a part of the analysis of a serial composition, somewhat analogous to identifying the various tonalities of a tonal work. Questions regarding form, thematic relationships, texture, rhythm, and other matters are just as relevant here as in the analysis of more traditional music. The music of classical serialism is not especially "mathematical," and it is not composed mechanically and without regard to the resulting sound or the effect on the listener. Probably the best way to appreciate the processes and choices involved in serial composition is to try to compose a good serial piece. The exercises at the end of this chapter will provide some practice at attempting this.
SUMMARY

The pitch materials of a serial work are derived from the twelve-note row, so an analysis should begin with the row itself. Two special types of row are derived sets and all-interval sets. A composition may make use of the prime row, its retrograde, its inversion, and its retrograde inversion, each of which can appear at any of twelve transpositions. This pitch material may be conveniently displayed in the form of a matrix.

Row forms may be used compositionally in a number of ways. For example, a single row form may be distributed among the voices, or more than one row form may be used at the same time. The choice of row forms is often related to invariance or combinatoriality, among other reasons.

Analysis of serial music includes identification of the row forms and consideration of the reasons for choosing a particular row form and transposition, but a thorough analysis cannot be confined only to serial matters.

NOTES


2. Another approach (and one that was used in previous editions of this book) labels the first appearance of the row as P-0, no matter what pitch class it begins with, and numbers the transpositions chromatically from that pitch class. Also, some authors have employed the letters S or O instead of P for the prime form of the row.

3. Another type of all-interval row contains all of the intervals only if some of them are ascending and others are descending. The Dallapiccola row discussed in this chapter is one example.


EXERCISES

Part A: Fundamentals

1. Suppose P-7 begins on G and ends on B♭:

<table>
<thead>
<tr>
<th>Form</th>
<th>Begins on</th>
<th>Ends on</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) P-6</td>
<td>___</td>
<td>___</td>
</tr>
<tr>
<td>(b) P-11</td>
<td>___</td>
<td>___</td>
</tr>
<tr>
<td>(c) R-0</td>
<td>___</td>
<td>___</td>
</tr>
<tr>
<td>(d) R-5</td>
<td>___</td>
<td>___</td>
</tr>
</tbody>
</table>