INTRODUCTION

In the first decade of the twentieth century, a few composers developed an approach to composition that, in retrospect, was perhaps inevitable. The chromaticism of the nineteenth century had chipped away at the tonal system so successfully that it was only a natural outcome for the system eventually to be abandoned altogether. This new music without a tonal center eventually became known as “atonal” music, although not without objection by some of the composers who originated the style.

Atonality is one of the more important aspects of twentieth-century music, and it is a major factor that distinguishes much of the music of this century from any other music in the Western tradition. Nonserial, or “free,” atonality led in the 1920s to a more organized atonal method called serialism or twelve-tone music, but nonserial atonal music continued to be composed and is still being composed today. We will discuss serialism in later chapters. For now we are concerned only with nonserial atonal music, which, for the present, we will refer to simply as atonal music.

It is not surprising that the analysis of atonal music has required the development of new theoretical terms and approaches. Although analytical methods are still being developed and experimented with, most of the current literature derives from the work of Allen Forte, whose book The Structure of Atonal Music is a standard reference on the subject.1 Most of the information in this chapter derives from Forte’s work or from that of his followers. Although the basics of this theory may be totally unfamiliar to you, the concepts are not very difficult, and you will find them applicable to a wide range of twentieth-century styles.
CHARACTERISTICS OF ATONAL MUSIC

There are several characteristics of atonal music that set it apart from other styles. The first of course, is that it lacks a tonal center. This aspect is a subjective one, for any two listeners might differ concerning the degree to which a tonal center is audible in a particular work; nevertheless, a great many pieces are widely accepted as being atonal. This atonality is achieved by avoiding the conventional melodic, harmonic, and rhythmic patterns that help to establish a tonality in traditional tonal music. In their place we find unresolved dissonances, a preponderance of mixed-interval chords, and pitch material derived from the chromatic scale. Textures often are contrapuntal, with themes or melodies in the traditional sense occurring less often, and the metric organization is frequently difficult for the listener to follow.

Example 9–1 is from the beginning of one of the earliest atonal works. Although it is not the most representative example that could have been chosen, it does have the advantage of being easy to play, and you will get more out of the following discussion if you play the example several times before going on. The piece opens with a slow tremolo between D and F over a sustained E♭, perhaps suggesting D minor. (Octave doublings such as the octave here on F were soon discarded by some atonal composers on the theory that they put too much emphasis on a single pitch class.) The melody that enters in m. 2 does little to confirm D as a tonality. Two short melodic phrases, one beginning in m. 2 and the other in m. 3, use the pitch classes C, D♭, D, E♭, A♭, and A, with many of these pitches being freely dissonant against the accompaniment. Following the rest under the fermata, a third phrase interrupts the tremolo figure in the low register and closes the excerpt. Melodically this third phrase begins with an expressive motion up to D (notice the accompanying chords here do not suggest a D tonal center), followed by the same pitch classes that ended the second phrase: C–E♭–A♭. The tremolo figure returns as the melody settles in on its last two notes. Whereas in this excerpt the D/F dyad (pair of notes) in the accompaniment is clearly in opposition to the D♭ in the melody, neither D nor D♭ is strongly established as a tonal center, and this, along with the prevailing chromaticism, leads us to classify this music as atonal rather than polytonal (review Chapter 5).
EXAMPLE 9–1  Schoenberg: Three Piano Pieces (1909), Op. 11, No. 2, mm. 1–5
(Used by permission of Belmont Music Publishers.)

It is doubtful that the listener would be able to identify the time signature used in these opening measures. The opening undulating figure is notated in groups of three notes but contains only two pitches, so the listener is not sure whether the first six eighth-notes constitute two or three beats (assuming the eighth-note is heard as a division of the beat). The melody in its first phrase works out nicely in \( \frac{3}{8} \), but, to express the meter as notated, the melody should start at the beginning of the measure instead of on the third eighth-note (Example 9–2a). The melody of the second phrase could easily be heard as two measures of \( \frac{3}{4} \) with an anacrusis (Example 9–2b). Later portions of the piece (not shown) suggest \( \frac{3}{4} \), \( \frac{3}{4} \), and \( \frac{3}{4} \), among others, although there are passages that are clearly in \( \frac{3}{8} \).

EXAMPLE 9–2  The Melody Renotated

a. b.
PITCH-CLASS SETS

It soon became clear to musicians that the pitch aspect of atonal music required a new vocabulary if the analysis of this music was ever to be more than descriptive. It was recognized that atonal music often achieved a certain degree of unity through recurrent use of a new kind of motive. This new kind of motive was given various names, including cell, basic cell, set, pitch set, pitch-class set, and referential sonority. It could appear melodically, harmonically, or as a combination of the two. The set also could be transposed and/or inverted (that is, in mirror inversion; for example, G–B–C inverts to G–Eb–D), and its pitches could appear in any order and in any register. Most pieces were found to employ a large number of different kinds of pitch sets, only a few of which might be important in unifying the piece. The analysis of atonal music usually includes the process of identifying and labeling these important pitch sets, a process that involves segmentation.

Segmentation is in some ways much more difficult than the analysis of chords in traditional tonal music. The first problem is that, when beginning the analysis, one usually does not know which sets will turn out to be significant in the piece and which ones will not, meaning that various musically convincing segmentations may have to be tried and discarded before the significant ones appear. We will demonstrate the process of segmentation throughout this chapter.

A second problem is labeling the sets for ease of comparison, and it is in this area that Allen Forte's work has proved so helpful. Because an atonal chord or melodic fragment can consist of any combination of pitches, thousands of different sets are possible. As we shall see, Forte's system of pitch-class sets reduces this number considerably.

OCTAVE EQUIVALENCE, TRANSPPOSITIONAL EQUIVALENCE, AND NORMAL ORDER

In the analysis of tonal music, we routinely reduce sonorities to basic forms. For instance, through the theories of octave equivalence and chord roots we analyze all of the chords in Example 9–3a as C major triads and all of those in Example 9–3b as F major triads. In addition, we consider C major triads and F major triads to be transpositionally equivalent, to be members of a class of sonorities referred to collectively as major triads. These concepts are so obvious to us that it seems trivial to mention them, but in fact the theory that classifies the sonorities in Example 9–3 into one chord type is only a few centuries old.

EXAMPLE 9–3 Major Triads

\[
\begin{array}{c}
\text{a.} \\
\ \hdashline \\
\text{b.} \\
\end{array}
\]

In order to analyze and compare the pitch-class sets in atonal music, we need a process that will reduce any set to some basic form in the same way that we reduce the
chords in Example 9–3 to the three notes of the major triad in root position. This basic form in pitch-set analysis is called the normal order. The procedure to follow in determining the normal order will be illustrated by means of the segmentations shown in Example 9–4. All of the segmentations are circled except for Sets 3 and 4; these sets are the three-note chords that accompany the melody.

**EXAMPLE 9–4** Schoenberg: Three Piano Pieces (1909) Op. 11, No. 2, mm. 1–5
(Used by permission of Belmont Music Publishers.)

The normal order of a pitch-class set is that ordering of the pitches that spans the smallest possible interval. In that sense, it is similar to putting a triad into root position. Some instructors prefer to use pitch-class integers (C = 0, C♯/Db = 1, etc.) when figuring out the normal order, but in this text we will use musical notation.

To find the normal order of a set, notate the pitches as an ascending “scale,” within a single octave, as in Example 9–5a. You can begin the scale on any one of the notes, and the notes may be spelled enharmonically if you wish. Leave out any duplicated pitch classes, but continue the scale up to the octave (the second Db in Example 9–5a). Now find the largest interval between any two adjacent notes (the tritone in Example 9–5a); the top note of this largest interval (the A in Example 9–5a) is the bottom note of the normal order. The normal order of set 1 is shown in Example 9–5b. Notice that in Example 9–5 we have labeled the interval A–Db as a major 3rd even though it is notated as a diminished 4th. While
enharmomic substitution is usually discouraged in tonal analysis, it is essential in analyzing atonal music. To write out a normal order without using musical notation, separate the note names with commas, and enclose the normal order between brackets, as in [A, D#, E].

\textbf{Example 9–5 Normal Order of Set 1}

When we put Set 2 into normal order in Example 9–6b, we discover that it is [A, C, D], the same as Example 9–5b, but a minor 2nd lower; in other words, Set 2 is transpositionally equivalent to Set 1.

\textbf{Example 9–6 Normal Order of Set 2}

Sets 3 and 4 are the three-note chords that accompany the melody in m. 4. The normal orders of these two sets are shown in Example 9–7. Notice that Set 4 [E, G, A], is transpositionally equivalent to Sets 1 and 2, but Set 3 is not.

\textbf{Example 9–7 Normal Orders of Sets 3 and 4}

Sets 5, 6, and 7 result from a different segmentation of m. 4. Each one contains four pitch classes. The first column of Example 9–8 shows these sets in scalar form. Notice that in Set 7 the G# and C have been spelled enharmonically for convenience. The normal orders of these sets are shown in the second column of Example 9–8. None of these sets is transpositionally equivalent to any of the others.
EXAMPLE 9–8 Normal Orders of Sets 5, 6, and 7

Set 5
\[ \begin{align*}
\text{M2} & \quad \text{m3} & \quad \text{m2} \\
\end{align*} \]

Set 6
\[ \begin{align*}
\text{M3} & \quad \text{M2} & \quad \text{P4} & \quad \text{m2} \\
\end{align*} \]

Set 7
\[ \begin{align*}
\text{M3} & \quad \text{m2} & \quad \text{P4} & \quad \text{M2} \\
\end{align*} \]

We are making good progress here in learning how to put pitch sets into normal order, which, you will recall, means that order that spans the smallest interval. A complication that occasionally arises is a set in which there is not a single largest interval, but instead two or more intervals are tied for largest. Set 8 is an example of such a set. Notice in Example 9–9a that there are two major 3rds and that all of the other intervals are smaller. This means that we have two candidates for the normal order, Example 9–9b and Example 9–9c. The tie is broken by comparing the intervals between the first and next-to-last notes in both versions (A–E♭ in Example 9–9b versus C♯–F in Example 9–9c). The normal order is the version with the smaller interval—Example 9–9c—because that version is the one that is packed most tightly to the left.

EXAMPLE 9–9 Normal Order of Set 8

If the intervals between the first and next-to-last notes in Example 9–9b and Example 9–9c had been the same, we would have proceeded to the intervals between the first and third-to-last notes, and so on, until the tie was broken. In some sets, however, the tie cannot be broken. Consider the set in Example 9–10a (not taken from the Schoenberg
It contains two instances of its largest interval, the minor 3rd, so there are two candidates for the normal order. These are shown in Example 9–10b and Example 9–10c. The interval successions in the two versions are identical (because they are transpositionally equivalent), so it is impossible to break the tie. In this case, either of the tied forms may be selected as the normal order. A set such as this is called a transpositionally symmetrical set, because it reproduces its own pitch-class content under one or more intervals of transposition. In the case of the set in Example 9–10, transposition at the tritone reproduces the set.

**EXAMPLE 9–10** A Transpositionally Symmetrical Set

![Diagram of a transpositionally symmetrical set with intervals shown]

**INVERSIONAL EQUIVALENCE AND BEST NORMAL ORDER**

We have seen that tonal and atonal analyses share the concepts of octave equivalence and transpositional equivalence. Atonal analysis goes a step further, however, and considers pitch-class sets that are related by inversion to be equivalent. (To “invert” a set in atonal music means to reverse the order of the intervals.)

This would not be a useful approach in tonal music, because the major and minor triads, for example, are related by inversion, as are the dominant 7th chord and the half-diminished-7th chord, and we need to be able to distinguish between them in tonal analysis. But in atonal music a set and its inversion are considered to be different representations of the same set class.

If we are going to have a single classification for any set and its inversion, then we will have to carry the concept of the normal order a step further, to something called the best normal order. This concept is important because the best normal order is the generic representation of all the possible transpositions and inversions of a set. In order to find the best normal order of any set, first find its normal order and then note its inversion. The inversion will already be in normal order unless there are two or more occurrences of the largest interval (discussed later). Finally, compare the two normal orders: the “better” of the two is considered to be the best normal order.

Let us see how this works with Set 1 from Example 9–4. Its normal order was given in Example 9–5. An easy way to invert the normal order of a set is to use the same top and bottom notes, and then fill in the remaining notes by reversing the order of the intervals. In Example 9–11a the intervals of the normal order are analyzed. Then, keeping the outer notes the same, these intervals are reversed in Example 9–11b to form the inversion. Finally, we choose between the two orders by comparing the intervals between the first and next-to-last notes in both versions, just as we did with Example 9–9. We select as the best normal order the version with the smaller interval—in this case, Example 9–11b.
EXAMPLE 9–11  Set 1 and Its Inversion

Looking back over Examples 9–5, 9–6, and 9–7, you can see that Sets 1, 2, and 4 were transpositionally equivalent to one another, and that all three were inversionally equivalent to Set 3. In other words, all four sets have the same best normal order and therefore are all representatives of the same set class.

Set 5 is analyzed in Example 9–12. The normal order of the original set is in Example 9–12a, and its inversion is in Example 9–12b. When the two normal orders, Example 9–12a and 9–12b, are compared, we see that the major 3rd (diminished 4th) in Example 9–12b is smaller than the perfect 4th in Example 9–12a, so Example 9–12b is selected as the best normal order. If the intervals between the first and next-to-last notes in Example 9–12a and Example 9–12b had been the same, we would have proceeded to the intervals between the first and third-to-last notes, and so on, until the tie was broken.

EXAMPLE 9–12  Best Normal Order of Set 5

In Example 9–13 Set 6 is analyzed. The normal order of the original set, Example 9–13a, turns out to be the best normal order.

EXAMPLE 9–13  Best Normal Order of Set 6

Set 7 is analyzed in Example 9–14. The best normal order is Example 9–14b. Because the best normal orders of Sets 6 and 7 (Example 9–13a and Example 9–14b) are transpositionally equivalent, Sets 6 and 7 are representatives of the same set class.
EXAMPLE 9–14  Best Normal Order of Set 7

It is not uncommon to find a set in which the normal order of the set and the normal order of its inversion are identical or transpositionally equivalent. An example is Set 9, shown in normal order in Example 9–15a. In Example 9–15b the normal order is inverted, and we see that the normal order of the inversion is the same as the original. A set such as this is called an inversionally symmetrical set because it reproduces its pitch-class content at one or more levels of inversion.

EXAMPLE 9–15  Best Normal Order of Set 9

The method that we have been using so far to find the best normal order will not always work with pitch-class sets that have more than one occurrence of the largest interval (such as the one in Example 9–9). Consider the set in Example 9–16a. There are two occurrences of the largest interval, the major 3rd, so we have to consider the two candidates for the normal order in Examples 9–16b and 9–16c. The interval between the first and next-to-last notes in Example 9–16b is smaller than that in Example 9–16c, so the normal order is [E, G, A♭, C].

EXAMPLE 9–16  A Pitch-Class Set

The normal order of this set is given again in Example 9–17a along with its inversion in 9–17b. However, because we know that there are two occurrences of the largest interval (the major 3rd), we have to consider another ordering of the inversion, shown in Example
9–17c. Comparing our three candidates for best normal order, we see that two of them span a major 3rd between the first and next-to-last notes, which eliminates Example 9–17b from the competition. We now back up one interval and compare the intervals between the first and third-to-last notes in Examples 9–17a (minor 3rd) and 9–17c (minor 2nd), looking for the smaller interval. As it turns out, the last version of our set wins out over the other two and is in fact the best normal order. The lesson here is that you must try out as many orderings of the inversion as there are occurrences of the largest interval or you may not discover the best normal order.

EXAMPLE 9–17  The Normal Order Inverted

PRIME FORMS AND SET CLASSES

Once we know the best normal order of a set, we need to be able to give it a name, which is done by applying numbers to the best normal order. The resulting series of numbers is called the prime form, and it represents all of the pitch-class sets in that set class, just as “major triad” represents all possible major triads in all possible arrangements. The first number of a prime form is always 0, and it stands for the lowest note of the best normal order. The other numbers give the distance in half steps each successive note is above that lowest note. For instance, in Example 9–11b we notated the best normal order of Set 1 as [A, B, Eb]. Because B is two half steps above A and Eb is six half steps above A, the name of this set class is [026], and [026] represents all transpositions and/or inversions of Set 1. Sets 1, 2, 3, and 4 are all [026] trichords. Notice that the set class is enclosed in brackets and that the numbers are not separated by commas or spaces. In the event that you need a 10 or 11 in a set name, use T or E; a whole tone scale would be [02468T].

A few more illustrations: The best normal order of Set 5 (Example 9–12b) is [E, F, Ab, Bb], yielding [0146] as a prime form. The best normal order of Set 6 (Example 9–13a) is [D, Eb, G, A], so its prime form is [0157], and Set 7 (Example 9–14b) is also a [0157] tetrachord. Set 9 (Example 9–15a) is a [0145] tetrachord.

By adopting the concepts of transpositional and inversive equivalence, the thousands of possible pitch combinations have been reduced to a manageable number of prime forms or set classes. The following table shows how many set classes there are for combinations of from two to ten pitch classes.
186 Nonserial Atonality

6 Dyads (2 pitch classes)
12 Trichords (3 pitch classes)
29 Tetrachords (4 pitch classes)
38 Pentachords (5 pitch classes)
50 Hexachords (6 pitch classes)
38 Septachords (7 pitch classes)
29 Octachords (8 pitch classes)
12 Nonachords (9 pitch classes)
6 Decachords (10 pitch classes)

220 TOTAL

THE INTERVAL-CLASS VECTOR

Most of pitch-set analysis is concerned with identifying sets that recur in a piece in compositionally significant ways. This includes, of course, transpositions and inversions of the original set, since we recognize transpositional and inversional equivalence. But analytical theory is much less advanced when it comes to comparing nonequivalent sets. Consider, for example, Sets 5 and 6 from the Schoenberg excerpt, reproduced here in best normal order beginning on G:

EXAMPLE 9–18—Sets 5 and 6

\[ [0146] \quad [0157] \]

It would appear that [0146] and [0157] are very similar, the only difference being the interval between the second and third notes; thus, [0157] is a kind of expansion of [0146]. But there are other differences, one of them being that [0157] contains two perfect intervals (G–C and G–D), while [0146] contains only one (Ab–C). This may mean that [0157] is potentially a more consonant sound than [0146]. One way of comparing sets that contain the same number of pitch classes, as these do, is to tabulate their interval contents. Because inversional equivalence is still in effect, we will consider the minor 2nd and the major 7th to be the same interval, also the major 2nd and the minor 7th, and so forth. We then have six interval classes ("interval class" is sometimes abbreviated as "IC"):

<table>
<thead>
<tr>
<th>Interval</th>
<th>Traditional</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>m2, M7</td>
</tr>
<tr>
<td>2</td>
<td>M2, m7</td>
</tr>
<tr>
<td>3</td>
<td>m3, M6</td>
</tr>
<tr>
<td>4</td>
<td>M3, m6</td>
</tr>
<tr>
<td>5</td>
<td>P4, P5</td>
</tr>
<tr>
<td>6</td>
<td>A4, d5</td>
</tr>
</tbody>
</table>
**EXERCISES**

**Part A: Fundamentals**

1. Analyze the pitches in each exercise below as a single pitch-class set. Notate the set in its normal order and in its best normal order (which may or may not be different), and write the prime form (e.g., \([0157]\)) underneath the best normal order.

2. Provide the interval-class vector for each set in Exercise A.1.

3. Classify each set from Exercise A.1 as (1) transpositionally symmetrical, (2) inversionally symmetrical, (3) both transpositionally and inversionally symmetrical, or (4) neither transpositionally nor inversionally symmetrical.

4. Notate the four trichordal subsets that can be derived from Exercise A.1.f. Notate them in normal order, not in best normal order. Which two belong to the same set class?

**Part B: Analysis**

1. See Example 3–15 (p. 54). What trichord type (prime form) is featured in the piano?

2. See Example 3–21 (p. 59). What trichord type (prime form) is featured in this passage from a tonal composition by Debussy?

3. See Example 3–27 (p. 62). What is the prime form of this hexachord? Is this set class transpositionally symmetrical, inversionally symmetrical, or both?

4. See Example 3–29 (p. 64). The same two tetrachord types appear in each measure on beats 1 and 2. What are the prime forms of these tetrachords? Are either or both of them transpositionally and/or inversionally symmetrical?

5. See Example 3–B–8 (p. 72). We saw in connection with Example 9–19 that the music alternates between \([0137]\) and \([0146]\) in the first two measures. What trichord type is found in the right hand in those measures? What set class ends the piece?

6. See Example 4–4 (p. 79). What tetrachord begins the flute part? The voice part? Where is the first aggregate completed? Is this a structurally significant location, such as a climactic pitch or the end of a phrase?

7. See Example 4–6 (p. 81). What trichord type appears under each of the three phrase marks?